SECI 1113
COMPUTATIONAL MATHEMATICS

ASSIGNMENT 3

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1. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix}$

$T(0) = \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2(0) - 0 \\ 0 + 3(0) \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\therefore$ Since $T(0) = 0$, therefore function T is linear transformation.

2. let $u = (x_1, y_1, z_1)$, $v = (x_2, y_2, z_2)$

$T(u) + T(v) = T\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + T\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$

$= \begin{bmatrix} x_1 + y_1 - z_1 \\ 2x_1 y_1 \\ x_1 + z_1 + 1 \end{bmatrix} + \begin{bmatrix} x_2 + y_2 - z_2 \\ 2x_2 y_2 \\ x_2 + z_2 + 1 \end{bmatrix}$

$= \begin{bmatrix} x_1 + x_2 + y_1 + y_2 - z_1 - z_2 \\ 2x_1 y_1 + 2x_2 y_2 \\ x_1 + x_2 + z_1 + z_2 + 2 \end{bmatrix}$

$T(u + v) = T\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}\right)$

$= T\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \\ z_1 + z_2 \end{bmatrix}$

$= \begin{bmatrix} x_1 + x_2 + y_1 + y_2 - z_1 - z_2 \\ 2x_1 y_1 + 2x_1 y_2 + 2x_2 y_1 + 2x_2 y_2 \\ x_1 + x_2 + z_1 + z_2 + 1 \end{bmatrix}$

$T(u) + T(v) \neq T(u + v)$

$$AT(u) = A\,T\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} \qquad T(Au) = T\begin{bmatrix} Ax_1 \\ Ay_1 \\ Az_1 \end{bmatrix}$$

$$= A\begin{bmatrix} x_1+y_1-z_1 \\ 2x_1y_1 \\ x_1+z_1+1 \end{bmatrix} \qquad = \begin{bmatrix} Ax_1+Ay_1-Az_1 \\ 2A^2x_1y_1 \\ Ax_1+Az_1+1 \end{bmatrix}$$

$$= \begin{bmatrix} Ax_1+Ay_1-Az_1 \\ 2Ax_1y_1 \\ Ax_1+Az_1+A \end{bmatrix}$$

$AT(u) \neq T(Au)$

$\therefore$ Since $T(u)+T(v) \neq T(u+v)$ and $AT(u) \neq T(Au)$, therefore it is not a linear transformation between vector space.

3 a.) $T(e_1) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ $T(e_2) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ $T(e_3) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$= \begin{bmatrix} x_1+2y_1+3z_1 \\ 2x_1+y_1+3z_1 \\ x_1+3y_1+2z_1 \end{bmatrix}$ $= \begin{bmatrix} x_2+2y_2+3z_2 \\ 2x_2+y_2+3z_2 \\ x_2+3y_2+2z_2 \end{bmatrix}$ $= \begin{bmatrix} x_3+2y_3+3z_3 \\ 2x_3+y_3+3z_3 \\ x_3+3y_3+2z_3 \end{bmatrix}$

b.) $3e_1 - 4e_2 + 6e_3$

$$= \begin{bmatrix} 3x_1+6y_1+9z_1-4x_2-8y_2-12z_2+6x_3+12y_3+18z_3 \\ 6x_1+3y_1+9z_1-8x_2-4y_2-12z_2+12x_3+6y_3+18z_3 \\ 3x_1+9y_1+6z_1-4x_2-12y_2-8z_2+6x_3+18y_3+12z_3 \end{bmatrix}$$

$T(3e_1 - 4e_2 + 6e_3)$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 3x_1+6y_1+9z_1-4x_2-8y_2-12z_2+6x_3+12y_3+18z_3 \\ 6x_1+3y_1+9z_1-8x_2-4y_2-12z_2+12x_3+6y_3+18z_3 \\ 3x_1+9y_1+6z_1-4x_2-12y_2-8z_2+6x_3+18y_3+12z_3 \end{bmatrix}$$

$$= \begin{bmatrix} 24x_1+39y_1+45z_1-32x_2-52y_2-60z_2+48x_3+102y_3+90z_3 \\ 21x_1+42y_1+45z_1-28x_2-58y_2-57z_2+42x_3+84y_3+108z_3 \\ 27x_1+33y_1+48z_1-36x_2-44y_2-64z_2+54x_3+66y_3+96z_3 \end{bmatrix}$$

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4. $T\left(\begin{bmatrix}1\\3\end{bmatrix}\right) = \begin{bmatrix}1-2(3)\\-2(1)+4(2)\end{bmatrix}$

$= \begin{bmatrix}-5\\6\end{bmatrix}$

$\therefore$ Since vector $v = \begin{bmatrix}1\\3\end{bmatrix} \neq 0$, therefore vector v is not in $N(T)$.

5. a.) $C_1\begin{bmatrix}-1\\2\\1\end{bmatrix} + C_2\begin{bmatrix}0\\5\\0\end{bmatrix} + C_3\begin{bmatrix}-1\\-1\\2\end{bmatrix} = \begin{bmatrix}-2\\1\\2\end{bmatrix}$

$\left[\begin{array}{ccc|c}-1 & 0 & -1 & -2\\2 & 5 & -1 & 1\\1 & 0 & 2 & 2\end{array}\right]$ $R_1 \to R_1 \div -1$

$\left[\begin{array}{ccc|c}1 & 0 & 1 & 2\\2 & 5 & -1 & 1\\1 & 0 & 2 & 2\end{array}\right]$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$\left[\begin{array}{ccc|c}1 & 0 & 1 & 2\\0 & 5 & -3 & -3\\0 & 0 & 1 & 0\end{array}\right]$

$C_3 = 0$ #

$5C_2 - 3(0) = -3$

$5C_2 = -3$

$C_2 = -3/5$ #

$C_1 + (0) = 2$

$C_1 = 2$ #

$\therefore$ W is in the range of T as it has a solution where $C_1 = 2$, $C_2 = -3/5$ and $C_3 = 0$.

b.) $\begin{bmatrix}-1 & 0 & -1\\2 & 5 & -1\\1 & 0 & 2\end{bmatrix}$ $R_1 \to R_1 \div -1$

$\begin{bmatrix}1 & 0 & 1\\2 & 5 & -1\\1 & 0 & 2\end{bmatrix}$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$\begin{bmatrix}1 & 0 & 1\\0 & 5 & -3\\0 & 0 & 1\end{bmatrix}$ $R_2 \to R_2 \div 5$

$\begin{bmatrix}1 & 0 & 1\\0 & 1 & -3/5\\0 & 0 & 1\end{bmatrix}$ $R_1 \to R_1 - R_3$, $R_2 \to R_2 + \frac{3}{5}R_3$ $\longrightarrow$ $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$

$R(T) = \text{Span}\left\{\begin{bmatrix}-1\\2\\1\end{bmatrix}, \begin{bmatrix}0\\5\\0\end{bmatrix}, \begin{bmatrix}-1\\-1\\2\end{bmatrix}\right\}$

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c.) $\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right]$ $R_1 \to R_1 \div -1$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right]$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - R_1$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 5 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $R_2 \to R_2 \div 5$

$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -3/5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$ $R_1 \to R_1 - R_3$, $R_2 \to R_2 + \frac{3}{5}R_3$

$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$

$\therefore$ No basis for $N(T)$ as $\dim N(T) = 0$

6.) a.) $T\left(\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$ $T\left(\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$ $T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$a_1\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a_2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + a_3\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$ $b_1\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + b_2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + b_3\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$-a_3 = 3$ $a_2 + 3 = 0$ $a_1 + 3 = 2$ $-b_3 = -1$ $b_2 - 1 = 0$ $b_1 - 1 = 0$

$a_3 = -3$ $a_2 = -3$ $a_1 = -1$ $b_3 = 1$ $b_2 = 1$ $b_1 = 1$

$c_1\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_3\begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ Matrix $= \begin{bmatrix} -1 & 1 & 1 \\ -3 & 1 & -1 \\ -3 & 1 & 2 \end{bmatrix}$

$-c_3 = 2$ $c_2 + 2 = 1$ $c_1 + 1 = 2$

$c_3 = -2$ $c_2 = -1$ $c_1 = 1$

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b.)
$$T\left(\begin{bmatrix}-2\\1\\3\end{bmatrix}\right)=\begin{bmatrix}1\\4\\4\end{bmatrix}$$

$$a_1\begin{bmatrix}-1\\1\\1\end{bmatrix}+a_2\begin{bmatrix}-1\\-1\\1\end{bmatrix}+a_3\begin{bmatrix}0\\1\\1\end{bmatrix}=\begin{bmatrix}-2\\1\\3\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}-1&-1&0&-2\\1&-1&1&1\\1&1&1&3\end{array}\right] R_1\to R_1\div -1$$

$$\left[\begin{array}{ccc|c}1&1&0&2\\1&-1&1&1\\1&1&1&3\end{array}\right] \begin{matrix}R_2\to R_2-R_1\\R_3\to R_3-R_1\end{matrix}$$

$$\left[\begin{array}{ccc|c}1&1&0&2\\0&-2&1&-1\\0&0&1&1\end{array}\right] R_2\to R_2\div -2$$

$$\left[\begin{array}{ccc|c}1&1&0&2\\0&1&-1/2&1/2\\0&0&1&1\end{array}\right] R_1\to R_1-R_2$$

$$\left[\begin{array}{ccc|c}1&0&1/2&3/2\\0&1&-1/2&1/2\\0&0&1&1\end{array}\right] \begin{matrix}R_1\to R_1-\frac{1}{2}R_3\\R_2\to R_2+\frac{1}{2}R_3\end{matrix}$$

$$\left[\begin{array}{ccc|c}1&0&0&1\\0&1&0&1\\0&0&1&1\end{array}\right]$$

$a_1=1,\ a_2=1,\ a_3=1$

$$\begin{bmatrix}-2\\1\\3\end{bmatrix}_B=\begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\begin{bmatrix}-1&1&1\\-3&1&-1\\-3&1&-2\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}1\\-3\\-4\end{bmatrix}$$

$$T(v)=1\begin{bmatrix}0\\0\\1\end{bmatrix}-3\begin{bmatrix}1\\0\\-1\end{bmatrix}-4\begin{bmatrix}-1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\4\\4\end{bmatrix}$$