SECI 1113
COMPUTATIONAL MATHEMATICS

ASSIGNMENT 2

LECTURER:

Dr Razana Alwee

SUBMITTED BY:

Navinthra Rao A/L VenkataKumar A20EC0104

Shahril Bin Saiful Bahri A20EC0144

Woon Zi Jian A20EC0171

Semester 2 2020/2021

NO: Assignment 2 Part 1 DATE:

1. let $u = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$, $v = \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$, $w = \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$

Axiom 3 :

$u+(v+w)$

$$= \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+(t_2+t_3) \\ 2-(t_2+t_3) \\ 3+(2t_2+2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(t_1+t_2+t_3) \\ 2-(t_1+t_2+t_3) \\ 3+(2t_1+2t_2+2t_3) \end{bmatrix}$$

$(u+v)+w$

$$= \begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix} + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(t_1+t_2+t_3) \\ 2-(t_1+t_2+t_3) \\ 3+(2t_1+2t_2+2t_3) \end{bmatrix}$$

$\therefore$ Since $u+(v+w) = (u+v)+w$, therefore axiom 3 is satisfied.

Axiom 7 :

$p(u+v)$

$$= p\begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(pt_1+pt_2) \\ 2-(pt_1+pt_2) \\ 3+(2pt_1+2pt_2) \end{bmatrix}$$

$pu+pv$

$$= \begin{bmatrix} 1+pt_1 \\ 2-pt_1 \\ 3+2pt_1 \end{bmatrix} + \begin{bmatrix} 1+pt_2 \\ 2-pt_2 \\ 3+2pt_2 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(pt_1+pt_2) \\ 2-(pt_1+pt_2) \\ 3+(2pt_1+2pt_2) \end{bmatrix}$$

$\therefore$ Since $p(u+v) = pu+pv$, therefore axiom 7 is satisfied.

Axiom 8 :

$(p+q)u$

$$= (p+q)\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} = \begin{bmatrix} 1+(pt_1+qt_1) \\ 2-(pt_1+qt_1) \\ 3+(2pt_1+2qt_1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(p+q)t_1 \\ 2-(p+q)t_1 \\ 3+(p+q)2t_1 \end{bmatrix}$$

$pu+qu$

$$= \begin{bmatrix} 1+pt_1 \\ 2-pt_1 \\ 3+2pt_1 \end{bmatrix} + \begin{bmatrix} 1+qt_1 \\ 2-qt_1 \\ 3+2qt_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(pt_1+qt_1) \\ 2-(pt_1+qt_1) \\ 3+(2pt_1+2qt_1) \end{bmatrix}$$

Since $(p+q)u = pu+qu$, therefore axiom 8 is satisfied.

$\therefore$ V is a vector space.

NO: DATE:

2\. a.) let $p = \begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix}$, $q = \begin{bmatrix} a_2-b_2 & a_2 \\ b_2+c_2 & a_2-c_2 \end{bmatrix}$

$$p+q = \begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix} + \begin{bmatrix} a_2-b_2 & a_2 \\ b_2+c_2 & a_2-c_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1-b_1+a_2-b_2 & a_1+a_2 \\ b_1+c_1+b_2+c_2 & a_1-c_1+a_2-c_2 \end{bmatrix}$$

$$= \begin{bmatrix} a_1+a_2-b_1-b_2 & a_1+a_2 \\ b_1+b_2+c_1+c_2 & a_1+a_2-c_1-c_2 \end{bmatrix}$$

$\therefore$ $p+q$ is in S.

$$rp = r\begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix}$$

$$= \begin{bmatrix} r(a_1-b_1) & ra_1 \\ r(b_1+c_1) & r(a_1-c_1) \end{bmatrix}$$

rp is in S.

$\therefore$ Therefore, S is a subspace of $M_{2\times2}$.

b.) $\begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$

$a_1 = 3$

$3 - b_1 = 5$

$b_1 = -2$

$3 - c_1 = 3$

$c_1 = 0$

$\therefore$ since the $\begin{vmatrix} 5 & 3 \\ -2 & 3 \end{vmatrix}$ matched the values, therefore it is in S.

NO: DATE:

3. (a)

$$c_1\begin{bmatrix}1\\2\\3\end{bmatrix}+c_2\begin{bmatrix}-1\\3\\-1\end{bmatrix}+c_3\begin{bmatrix}1\\2\\-1\end{bmatrix}+c_4\begin{bmatrix}0\\6\\1\end{bmatrix}+c_5\begin{bmatrix}-3\\4\\5\end{bmatrix}=\begin{bmatrix}p\\q\\r\end{bmatrix}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&p\\2&3&2&6&4&q\\3&-1&-1&1&5&r\end{array}\right]\begin{array}{l}\\R_2=R_2-2R_1\\R_3=R_3-3R_1\end{array}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&p\\0&5&0&6&10&q-2p\\0&2&-4&1&14&r-3p\end{array}\right]\begin{array}{l}\\\\R_3=R_3-\frac{2}{5}R_2\end{array}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&p\\0&5&0&6&10&q-2p\\0&0&-4&-\frac{7}{5}&10&r-\frac{2}{5}q-\frac{11}{5}p\end{array}\right]$$

$\therefore$ p, q, r has a solution. Hence, span $(S)=R^3$.

(b)

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\2&3&2&6&4&0\\3&-1&-1&1&5&0\end{array}\right]\begin{array}{l}\\R_2=R_2-2R_1\\R_3=R_3-3R_1\end{array}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\0&5&0&6&10&0\\0&2&-4&1&14&0\end{array}\right]\begin{array}{l}\\\\R_3=R_3-\frac{2}{5}R_2\end{array}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\0&5&0&6&10&0\\0&0&-4&-\frac{7}{5}&10&0\end{array}\right]$$

$c_5=t$

$c_4=s$

$-4c_3-\frac{7}{5}s+10t=0$

$-4c_3=\frac{7}{5}s-10t$

$c_3=-\frac{7}{20}s+\frac{5}{2}t$

$5c_2+6s+10t=0$

$5c_2=-6s-10t$

$c_2=-\frac{6}{5}s-2t$

$c_1+(\frac{6}{5}s+2t)-\frac{7}{20}s+\frac{5}{2}t-3t=0$

$c_1=-\frac{17}{20}s-\frac{3}{2}t$

$\therefore$ Span $(S)=R^3$, but S is not linearly independent because S does not have a trivial solution. Therefore, S is not basis of R^3.

NO: DATE:

4. (a) $c_1\begin{bmatrix}1\\1\end{bmatrix}+c_2\begin{bmatrix}3\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

$\left[\begin{array}{cc|c}1&3&1\\1&-1&0\end{array}\right]$ $R_2=R_2-R_1$

$\left[\begin{array}{cc|c}1&3&1\\0&-4&-1\end{array}\right]$

$-4c_2=-1$ $\qquad$ $c_1+3(1/4)=1$

$c_2=1/4$ $\qquad$ $c_1=1/4$

b.) $c_1\begin{bmatrix}1\\-1\end{bmatrix}+c_2\begin{bmatrix}0\\2\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$ $\qquad$ $b_1\begin{bmatrix}1\\-1\end{bmatrix}+b_2\begin{bmatrix}0\\2\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$

$c_1=1$ $\qquad$ $b_1=3$

$-1+2c_2=1$ $\qquad$ $-3+2b_2=-1$

$2c_2=2$ $\qquad$ $2b_2=2$

$c_2=1$ $\qquad$ $b_2=1$

$[I]_{B_1}^{B_2}=\begin{bmatrix}1&3\\1&1\end{bmatrix}$

c) $[v]_{B_1}=\begin{bmatrix}1&3\\1&1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}$

$=\begin{bmatrix}11\\5\end{bmatrix}$