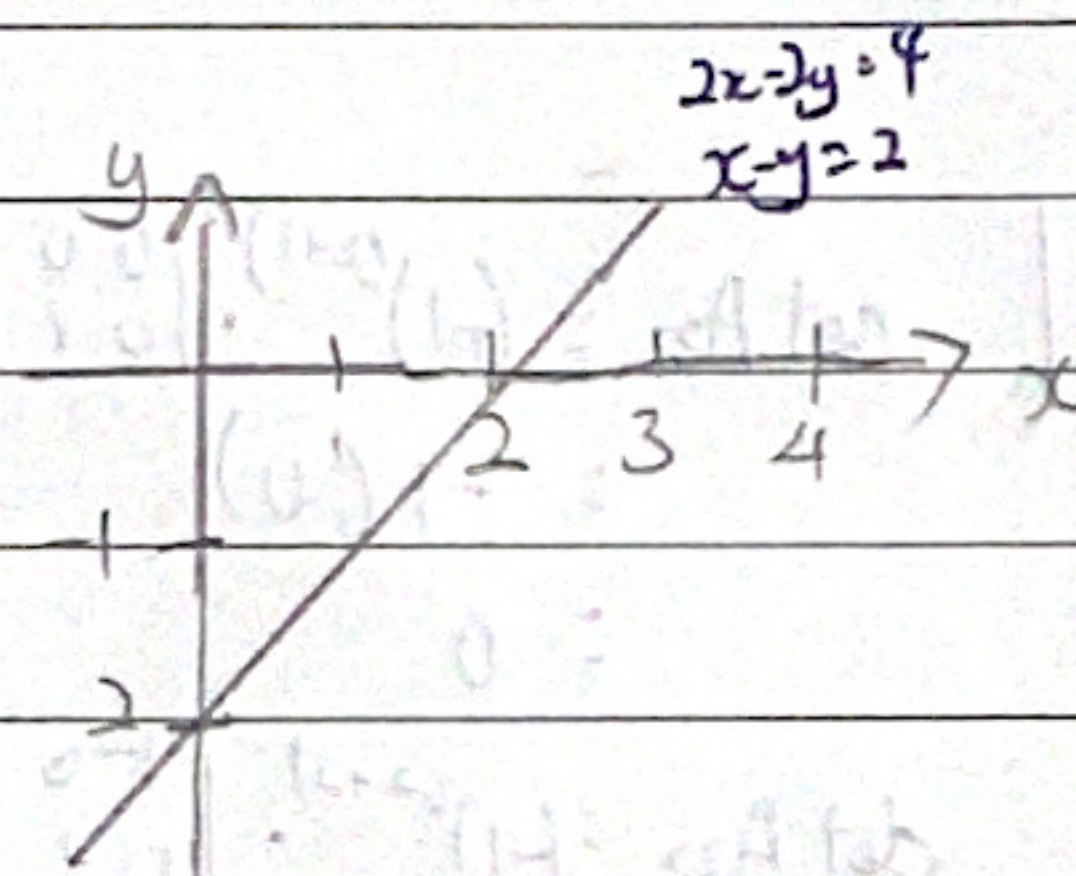


1. $2x - y = 3$
 $x + 3y = 5$
 when point $(1, -1)$,
 $2(1) - (-1) = 3$
 $(1) + 3(-1) = -2$
 $-2 \neq 5$

when point $(2, 1)$,
 $2(2) - (1) = 3$
 $(2) + 3(1) = 5$

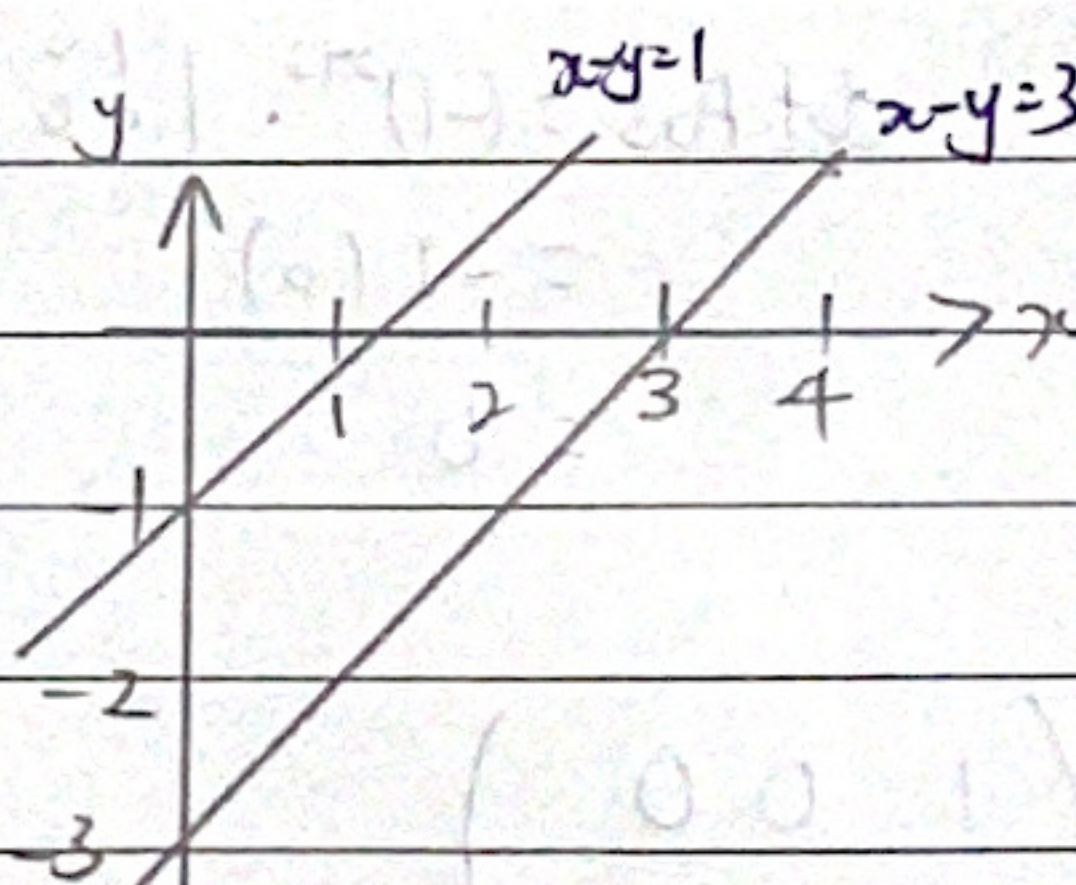
$(1, -1)$ is not a solution for the given systems of equation while $(2, 1)$ is the solution for the given systems.

2a) $x - y = 2$ - ①
 $2x - 2y = 4$ - ②



The second equation is equivalent to the first equation. It consists of infinite number of solutions.

b) $x - y = 1$ - ①
 $x - y = 3$ - ②



② - ①, $0 = 2$

$\therefore$ The system has no solution.

3a) $1 \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - 0 \begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} + 3 \begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$
 $= 1(2 - 1) - 0 + 3(5)$
 $= 16$

b) $1 \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - (-1) \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + 0 \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$
 $= 1(0 - 1) - (-1)(1 - 0) + 0$
 $= 0$

$$4a) A^{-1} = \frac{1}{(3 \times 1) - (0 \times 0)} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}$$

$$b) \det A = \begin{vmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1(1-0) - 0 + 0$$

$$= 1(1-0) - 0 + 0$$

$$= 1$$

$$\det A_{11} = (-1)^{1+1} \cdot \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} \quad \det A_{21} = (-1)^{2+1} \cdot \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} \quad \det A_{31} = (-1)^{3+1} \cdot \begin{vmatrix} 0 & 0 \\ 1 & -2 \end{vmatrix}$$

$$= 1(1-0)$$

$$= -1(0)$$

$$= 1(0)$$

$$= 1$$

$$= 0$$

$$= 0$$

$$\det A_{12} = (-1)^{1+2} \cdot \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} \quad \det A_{22} = (-1)^{2+2} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \quad \det A_{32} = (-1)^{3+2} \cdot \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix}$$

$$= -1(0)$$

$$= 1(1)$$

$$= -1(-2)$$

$$= 0$$

$$= 1$$

$$= 2$$

$$\det A_{13} = (-1)^{1+3} \cdot \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \quad \det A_{23} = (-1)^{2+3} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \quad \det A_{33} = (-1)^{3+3} \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 1(0)$$

$$= -1(0)$$

$$= 1(1)$$

$$= 0$$

$$= 0$$

$$= 1$$

$$(\text{Cofactor matrix } A)^T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{|A|} (\text{cofactor matrix } A)^T$$

$$= \frac{1}{1} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

5. a.) $\begin{bmatrix} 1 & 2 & -3 & | & 9 \\ 2 & -1 & 1 & | & 0 \\ 4 & -1 & 1 & | & 4 \end{bmatrix}$ $M_{21} = -\frac{2}{1} = -2$ $M_{31} = -\frac{4}{1} = -4$
 $R_2 \rightarrow R_2 + (-2)R_1$
 $R_3 \rightarrow R_3 + (-4)R_1$
 $\begin{bmatrix} 1 & 2 & -3 & | & 9 \\ 0 & -5 & 7 & | & -18 \\ 0 & -9 & 13 & | & -32 \end{bmatrix}$ $M_{32} = -\frac{9}{-5} = \frac{9}{5}$
 $R_3 \rightarrow R_3 + (-\frac{9}{5})R_2$
 $\begin{bmatrix} 1 & 2 & -3 & | & 9 \\ 0 & -5 & 7 & | & -18 \\ 0 & 0 & \frac{7}{5} & | & \frac{7}{5} \end{bmatrix}$ $\frac{7}{5}x_3 = \frac{7}{5}$ $-5x_2 + 7(1) = -18$ $x_1 + 2(5) + (-3)(1) = 9$
 $x_3 = 1$ $-5x_2 = -25$ $x_1 = 2$
 $x_2 = 5$
 $\therefore x_1 = 2, x_2 = 5, x_3 = 1$

b.) $\begin{bmatrix} 1 & 1 & 2 & | & 8 \\ -1 & -2 & 3 & | & 1 \\ 3 & -7 & 4 & | & 10 \end{bmatrix}$ $M_{21} = -\frac{-1}{1} = 1$ $M_{31} = -\frac{3}{1} = -3$
 $R_2 \rightarrow R_2 + (1)R_1$
 $R_3 \rightarrow R_3 + (-3)R_1$
 $\begin{bmatrix} 1 & 1 & 2 & | & 8 \\ 0 & -1 & 5 & | & 9 \\ 0 & -10 & -2 & | & -14 \end{bmatrix}$ $M_{32} = -\frac{-10}{-1} = -10$
 $R_3 \rightarrow R_3 + (-10)R_2$
 $\begin{bmatrix} 1 & 1 & 2 & | & 8 \\ 0 & -1 & 5 & | & 9 \\ 0 & 0 & -52 & | & -104 \end{bmatrix}$ $-52x_3 = -104$ $-x_2 + 5(2) = 9$ $x_1 + 1 + 2(2) = 8$
 $x_3 = 2$ $-x_2 = -1$ $x_1 = 3$
 $x_2 = 1$
 $\therefore x_1 = 3, x_2 = 1, x_3 = 2$

6a. $\begin{bmatrix} 1 & -3 & -2 & | & 0 \\ -1 & 2 & 1 & | & 0 \\ 2 & 4 & 6 & | & 0 \end{bmatrix}$ $R_2 \rightarrow R_2 + R_1$
 $R_3 \rightarrow R_3 - 2R_1$
 $\begin{bmatrix} 1 & -3 & -2 & | & 0 \\ 0 & -1 & -1 & | & 0 \\ 0 & 10 & 10 & | & 0 \end{bmatrix}$ $R_2 \rightarrow R_2 \div (-1)$ let $x_3 = t$
 $x_1 + t = 0$
 $x_1 = -t$
 $x_2 + t = 0$
 $x_2 = -t$
 $R_3 \rightarrow R_3 - 10R_2$
 $\begin{bmatrix} 1 & 0 & 1 & | & 0 \\ 0 & 1 & 1 & | & 0 \\ 0 & 0 & 0 & | & 0 \end{bmatrix}$ $\therefore x_1 = -t, x_2 = -t, x_3 = t$

$$b) \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{bmatrix} \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & -4 \end{bmatrix} R_3 \rightarrow R_3 + 4R_2$$

$$\begin{bmatrix} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{array}{l} R_1 \rightarrow R_1 + R_2 \\ R_2 \rightarrow R_2 \div (-1) \end{array}$$

$$\begin{bmatrix} 2 & 0 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} R_1 \rightarrow R_1 \div 2$$

$$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\therefore r = 2, s = -1$$

$$7a) \begin{bmatrix} 1 & 0 & 0 \\ 6_{21} & 1 & 0 \\ 3_1 & 6_{32} & 1 \end{bmatrix} \begin{bmatrix} u_1 & u_2 & u_3 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{23} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$u_{11} = 2, u_{12} = 1, u_{13} = 4$$

$$2u_{21} = 8 \quad 4(1) + u_{22} = 3 \quad 4(4) + u_{23} = 2$$

$$u_{21} = 4 \quad u_{22} = -7 \quad u_{23} = -14$$

$$2u_{31} = 4 \quad 2(1) - 7u_{32} = 11 \quad 2(4) - 14(-\frac{9}{7}) + u_{33} = -1$$

$$u_{31} = 2 \quad u_{32} = -\frac{9}{7} \quad u_{33} = -27$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 - \frac{9}{7} & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 - \frac{9}{7} & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$LY = \beta$$

$$y_1 = 12$$

$$y_2 = 20 - (48) = -28$$

$$y_3 = 33 - 24 + (\frac{9}{7})(-28) = -27$$

$$Y = [12 \ -28 \ -27]^T$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$$U \cdot X = Y$$

$$-27x_3 = -27 \quad -7x_2 - 14(1) = -28 \quad 2x_1 + 1(2) + 4(1) = 12$$

$$x_3 = 1$$

$$x_2 = 2$$

$$x_1 = 3$$

$$X = [3 \ 2 \ 1]^T$$

$$\therefore x=3, y=2, z=1$$

$$7b) \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$l_{11} = 2_{\#} \quad l_{21} = 8_{\#} \quad l_{31} = 4_{\#}$$

$$2u_{12} = 1 \quad 2u_{13} = 4$$

$$u_{12} = \frac{1}{2}_{\#} \quad u_{13} = 2_{\#}$$

$$\frac{1}{2}(8) + l_{22} = -3 \quad 8(2) - 7u_{23} = 2$$

$$l_{22} = -7_{\#}$$

$$u_{23} = 2_{\#}$$

$$4(\frac{1}{2}) + l_{32} = 11 \quad 4(2) + 9(2) + l_{33} = -1$$

$$l_{32} = 9_{\#}$$

$$l_{33} = -27_{\#}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

L

U

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

L

Y

=

B

$$2y_1 = 12$$

$$8(6) + (-7)y_2 = 20$$

$$4(6) + 9(4) + (-27)y_3 = 33$$

$$y_1 = 6_{\#}$$

$$y_2 = 4_{\#}$$

$$y_3 = \frac{1}{3}_{\#}$$

$$Y = [6 \ 4 \ \frac{1}{3}]^T$$

$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$$x_3 = 1 \quad x_2 + 2(1) = 4$$

$$x_1 + \frac{1}{2}(2) + 2(1) = 6$$

$$x_2 = 4 - 2(1) = 2$$

$$x_1 = 3$$

$$X = [3 \ 2 \ 1]^T$$

$$x=3, y=2, z=1$$

8. $d(u, v) = \|u - v\|$

$$= \sqrt{(1-0)^2 + (1-2)^2 + (-2)^2}$$

$$= \sqrt{2+1+9}$$

$$= \sqrt{12}$$

$$= 2\sqrt{3}$$

9. a.) $u \cdot v = 2(3) + 1(3) + 2(1)$

$$= 6 + 3 + 2$$

$$= 11$$

b) $u \times v = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix}, \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix}, \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}$

$$= 1-6, -(2-6), 6-3$$

$$= (-5, 4, 3)$$

$$(10a) \quad C_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + C_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{array} \right] \begin{array}{l} R_2 \rightarrow R_2 - R_1 \\ R_3 \rightarrow R_3 - R_1 \end{array}$$

$$5C_3 = 10 \quad C_2 + (-3)(2) = -3$$

$$C_3 = 2 \quad C_2 = 3$$

$$C_1 + 3 + (2) = 1$$

$$C_1 = -6$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{array} \right] R_3 \rightarrow R_3 - 2R_2$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 5 & 10 \end{array} \right]$$

$$-6 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + 3 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$$

$\therefore V$ in $\mathbb{R}^3$ is a linear combination of the following vectors.

$$b) \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 0 \\ 5 & 1 & 2 & 0 \end{array} \right] \begin{array}{l} R_2 \rightarrow R_2 - 2R_1 \\ R_3 \rightarrow R_3 - 5R_1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -9 & -3 & 0 \end{array} \right] \begin{array}{l} R_1 \rightarrow R_1 - 2R_2 \\ R_3 \rightarrow R_3 + 9R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 24 & 0 \end{array} \right] R_3 \rightarrow R_3 \div 24$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right] \begin{array}{l} R_1 \rightarrow R_1 + 5R_3 \\ R_2 \rightarrow R_2 - 3R_3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array} \right]$$

$$\therefore C_1 = 0, C_2 = 0, C_3 = 0$$

$$0 \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} + 0 \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} + 0 \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore u, v, \text{ and } w$ are linearly independent.