

A Q1. Assignment 1 #

$$A = \{1, 2\} \quad C = \{1, 4, 5, 6, 7, 8\}$$

$$B = \{1, 2, 3\}$$

$$a) A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$b) (A \cup B)' = A' \cap B'$$

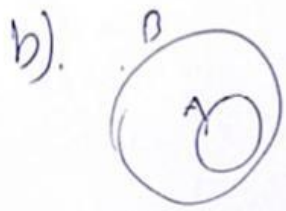
$$A' = U - A = \{3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$

$$B' = U - B = \{4, 5, 6, 7, 8, 9, 10, \dots\}$$

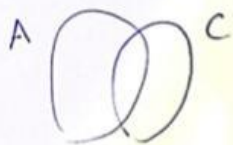
$$A' \cap B' = \{4, 5, 6, 7, 8, 9, 10, \dots\}$$

$$A' \cup B' = \{3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$

2:



$A \cap C \neq \emptyset$



a) Find t_7 .

$$\begin{aligned}t_7 &= t_6 + t_5 + t_4 \\ &= 13 + 7 + 4 = 24\end{aligned}$$

$$\begin{aligned}t_6 &= t_5 + t_4 + t_3 \\ &= 7 + 4 + 2 = 13\end{aligned}$$

$$\begin{aligned}t_5 &= t_4 + t_3 + t_2 \\ &= 4 + 2 + 1 = 7\end{aligned}$$

$$\begin{aligned}t_4 &= t_3 + t_2 + t_1 \\ &= 2 + 1 + 1 = 4\end{aligned}$$

$$\begin{aligned}t_3 &= t_2 + t_1 + t_0 \\ &= 1 + 1 + 0 = 2\end{aligned}$$

b) Write a recursive algorithm to compute t_n , $n \geq 3$.

Input: n positive integer

Output: $t(n)$

$t(n)$

$a_n = 3(a_{n-1} + a_{n-2})$, $n > 2$

7. If $f: \mathbf{R} \rightarrow \mathbf{R}$ and $g: \mathbf{R} \rightarrow \mathbf{R}$ are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

if $f(x_1) = f(x_2)$, then $x_1 = x_2$ Or, equivalently, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.
 Symbolically, $f: X \rightarrow Y$ is one-to-one $\iff x_1, x_2 \in X, \text{ if } f(x_1) = f(x_2) \text{ then } x_1 = x_2$.
 So yes it is one-to-one.

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer $n \geq 1$, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for c_1, c_2, \dots, c_n .

Given We can climb a staircase using 1 stair at a time or 2 stairs at a time or any combination of 1-stair and 2-stair steps

C_n = Number of different ways to climb a staircase with n stairs.

When $n =$

1, the staircase only contains 1 stair and thus we can only take the staircase by using 1 stair at a time once, which is exactly 1 way
 $= 1$

When $n = 2$ the staircase only contains 2 stairs. We can then take the 2 stairs at one or take the stairs one by one, which thus results in 2 different ways.

$C_2 = 2$

When $n > 3$, the staircase contains more than 2 stairs and thus we will need to use a combination of 1-stair and 2-stair steps

If the last move will be a 1-stair step, then there were $n - 1$ ways to arrive at the previous stair (which was a staircase with $n - 1$ stairs)

If the last move will be a 2-stair step, then there were $n - 2$ ways to arrive at the previous stair (which was a staircase with $n - 2$ stairs)

The total number of ways is then the sum of the number of ways in which the last move is a 1-stair step and the number of ways in which the last move is a 2-stair step.

$C_n = C_{n-1} + C_{n-2}$

$C_1 = 1, C_2 = 2, C_n = C_{n-1} + C_{n-2}$ when $n \geq 3$

9. The Tribonacci sequence (t_n) is defined by the equations,

$$t_0 = 0, t_1 = t_2 = 1, \quad t_n = t_{n-1} + t_{n-2} + t_{n-3} \quad \text{for all } n \geq 3.$$

0, 1, 1, 2, 4, 7, 13, 24, 44, 81, 148, 275, 507, 930, 1713, 3150, 5837, 10810, 20197, 37487, 69494, 128581, 238262, 442343, 825205, 1534850, 2842496, 5270544, 9778099, 18145645, 33766889, 62549529, 116092063, 215428497, 402916535, 751655107, 1395099745, 2593525887, 4819870737, 8955156569, 16615033201, 30861786557, 57476976553, 107448376350, 200699714710, 374268057617, 695538156927, 1294525980204, 2419972404848, 4511142748070, 8417043383125, 15672459756047, 29247735987272, 54337429166444, 101759627033863, 189169802957180, 353304660155487, 659644780186530, 1234127664079157, 2297150664422817, 4294974556268504, 8048770044212431, 15005793401553752, 27926016648035195, 51785512715891477, 96468235700679801, 180211476781932375, 335766839239714701, 624045143048667001, 1160189566496999007, 2167127066885981709, 4059973566812548717, 7580266600001431431, 14078165835849919150, 26310203065000000000, 48989665300000000000, 91711165300000000000, 171702030650000000000, 321711653000000000000, 599896653000000000000, 1117116530000000000000, 2089665300000000000000, 3917116530000000000000, 7317116530000000000000, 13696653000000000000000, 25696653000000000000000, 48171165300000000000000, 90171165300000000000000, 168966530000000000000000, 316966530000000000000000, 591711653000000000000000, 1091711653000000000000000, 2031653000000000000000000, 3769665300000000000000000, 7017116530000000000000000, 13016530000000000000000000, 24171165300000000000000000, 44696653000000000000000000, 83165300000000000000000000, 153165300000000000000000000, 281711653000000000000000000, 521653000000000000000000000, 961653000000000000000000000, 1769665300000000000000000000, 3269665300000000000000000000, 6017116530000000000000000000, 11016530000000000000000000000, 20171165300000000000000000000, 37165300000000000000000000000, 68165300000000000000000000000, 125165300000000000000000000000, 231711653000000000000000000000, 426966530000000000000000000000, 791653000000000000000000000000, 1451653000000000000000000000000, 2669665300000000000000000000000, 4916530000000000000000000000000, 9016530000000000000000000000000, 16696653000000000000000000000000, 30696653000000000000000000000000, 56165300000000000000000000000000, 103165300000000000000000000000000, 190165300000000000000000000000000, 351653000000000000000000000000000, 641653000000000000000000000000000, 1181653000000000000000000000000000, 2181653000000000000000000000000000, 4016530000000000000000000000000000, 7416530000000000000000000000000000, 13616530000000000000000000000000000, 25165300000000000000000000000000000, 46165300000000000000000000000000000, 85165300000000000000000000000000000, 156165300000000000000000000000000000, 286165300000000000000000000000000000, 531653000000000000000000000000000000, 981653000000000000000000000000000000, 1801653000000000000000000000000000000, 3316530000000000000000000000000000000, 6016530000000000000000000000000000000, 11016530000000000000000000000000000000, 20165300000000000000000000000000000000, 371653000000000000000000000000000000000, 681653000000000000000000000000000000000, 1251653000000000000000000000000000000000, 2316530000000000000000000000000000000000, 4216530000000000000000000000000000000000, 7716530000000000000000000000000000000000, 14165300000000000000000000000000000000000, 26165300000000000000000000000000000000000, 47165300000000000000000000000000000000000, 86165300000000000000000000000000000000000, 156165300000000000000000000000000000000000, 286165300000000000000000000000000000000000, 521653000000000000000000000000000000000000, 951653000000000000000000000000000000000000, 1716530000000000000000000000000000000000000, 3116530000000000000000000000000000000000000, 5616530000000000000000000000000000000000000, 10165300000000000000000000000000000000000000, 18165300000000000000000000000000000000000000, 331653000000000000000000000000000000000000000, 601653000000000000000000000000000000000000000, 1081653000000000000000000000000000000000000000, 1981653000000000000000000000000000000000000000, 36165300, 65165300, 118165300, 218165300, 401653000, 741653000, 1361653000, 25165300, 46165300, 85165300, 156165300, 286165300, 521653000, 951653000, 17165300, 31165300, 56165300, 10165300, 18165300, 33165300, 60165300, 1081653000, 1981653000, 36165300, 65165300, 1181653000, 2181653000, 40165300, 74165300, 1361653000, 25165300, 46165300, 85165300, 1561653000, 2861653000, 52165300, 95165300, 17165300, 31165300, 56165300, 10165300, 18165300, 33165300, 60165300, 1081653000, 1981653000, 36165300, 65165300, 1181653000, 2181653000, 40165300, 74165300, 1361653000, 25165300, 46165300, 85165300, 1561653000, 2861653000, 52165300, 95165300, 17165300, 31165300, 56165300, 10165300, 18165300, 33165300, 60165300, 1081653000, 1981653000, 36165300, 65165300, 1181653000, 2181653000, 40165300, 74165300, 1361653000, 25165300, 46165300, 85165300, 1561653000, 2861653000, 52165300, 95165300, 17165300, 31165300, 56165300, 10165300, 18165300, 33165300, 60165300, 1081653000, 1981653000, 36165300, 65165300, 1181653000, 2181653000, 40165300, 74165300, 1361653000, 25165300, 46165300, 85165300, 1561653000, 2861653000, 52165300, 95165300, 17165300, 31165300, 56165300, 10165300, 18165300, 33165300, 60165300, 1081653000, 1981653000, 36165300, 65165300, 1181653000, 2181653000, 40165300, 74165300, 1361653000, 25165300, 46165300, 85165300, 1561653000, 2861653000, 52165300, 95165300, 17165300, 31165300, 56165300, 10165300, 18165300,

$$(6) \quad \{1, 2, 3\}$$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \quad R_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$R_1 = \{ (1,1), (2,2), (2,3), (3,1), (3,3) \}$$

$$R_2 = \{ (1,2), (2,2), (3,1), (3,3) \}$$

$$(a) \quad R_1 \cup R_2 = \{ (1,1), (2,2), (2,3), (3,1), (3,3), (1,2) \}$$

$$(b) \quad R_1 \cap R_2 = \{ (2,2), (3,3), (3,1) \}$$

$$R_1 \cap R_2$$

$$(b) \quad \begin{matrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 3 & 0 & 1 \\ 3 & 1 & 0 \\ 3 & 1 & 1 \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$R_1 \cup R_2$$

$$(a) \quad \begin{matrix} 1 & 2 & 3 \\ 1 & 1 & 0 \\ 2 & 0 & 1 \\ 3 & 1 & 0 \\ 3 & 1 & 1 \end{matrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

(c) ① $R_1 =$ Not Reflexive because $(4,4) \notin R_1$
 $(5,5) \notin R_1$

② $R_1 =$ Symmetric $(1,2) (1,3) \in R_1$
 $(2,1) (3,1) \in R_1$

③ $R_1 =$ Is transitive ~~$(1,2) (2,3) (1,3) \in R_1$~~

$(3,2) (2,3) (3,3) \in R_1$

$R_1 =$ ~~Not~~ from ①, ②, ③ R_1 Not Equivalence

(d) $R_2 =$ Not reflexive $(1,1) (3,3) (5,5) \notin R_2$
 $(2,2) (4,4) \in R_2$

$R_2 =$ Not Symmetric $(1,2) \notin R_2$ but $(2,1) \in R_2$

$R_2 =$ transitive because $(4,2) (2,1) (4,1) \in R_2$

5

$$R = \{(x, y) \mid x + y \leq 6\}$$

$$R_1 = \{(y, z) \mid y \geq z\}$$

$$R_2 = y \rightarrow z$$

$$R_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$$

$$R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$$

6

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

6

$$\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

(4)

q	p	$\neg p$	$\neg q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$	$p \vee q$	$\neg(\neg(p \vee q))$	$(\neg p \vee \neg q) \vee (p \wedge q)$
T	T	F	F	F	T	T	T	T
T	F	T	F	T	F	T	F	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	F	F	F	T

$$S \cap T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ and } (x,y) \in T\}$$

$$S \cup T = \{(x,y) \in A \times B \mid (x,y) \in S \text{ or } (x,y) \in T\}$$

Let $A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$ and defined binary relations S and T from A to B as follows:

$$\text{For all } (x,y) \in A \times B, \quad x S y \leftrightarrow |x| = |y|$$

$$\text{For all } (x,y) \in A \times B, \quad x T y \leftrightarrow x - y \text{ is even}$$

State explicitly which ordered pairs are in $A \times B$, S , T , $S \cap T$, and $S \cup T$.

$$A \times B = \{(-1,1), (-1,2), (1,1), (1,2), (2,1), (2,2), (4,1), (4,2)\}$$

$$S = \{(-1,1), (1,1), (2,2)\}$$

$$T = \{(-1,1), (1,1), (2,2), (4,2)\}$$

$$S \cap T = \{(-1,1), (1,1), (2,2)\}$$

$$S \cup T = \{(-1,1), (1,1), (2,2), (4,2)\}$$

A Q₁.

Assignment 1 #

$$A = \{1, 2\} \quad C = \{3, 4, 5, 6, 7, 8\}$$

$$B = \{1, 2, 3\}$$

$$a) A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$$

$$b) (A \cup B)' = A' \cap B'$$

$$A' = U - A = \{3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$

$$B' = U - B = \{4, 5, 6, 7, 8, 9, 10, \dots\}$$

$$A' \cap B' = \{4, 5, 6, 7, 8, 9, 10, \dots\}$$

$$c) A' \cup B' = \{3, 4, 5, 6, 7, 8, 9, 10, \dots\}$$

Q₂.