A Q1. <u>Assigment 1 #</u>

$A = \{1, 2\}$ $C = \{3, 4, 5, 6, 7, 8\}$

$B = \{1, 2, 3\}$

a) $A \cup C = \{1, 2, 3, 4, 5, 6, 7, 8\}$

b) $(A \cup B)' = A' \cap B'$

$A' = U - A = \{3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

$' = U - B = \{4, 5, 6, 7, 8, 9, 10, \ldots\}$

~~AA~~ $A' \cap B' = \{4, 5, 6, 7, 8, 9, 10, \ldots\}$

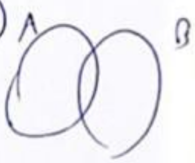
) $A' \cup B' = \{\cancel{\ldots}, 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

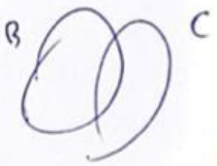
2

b).

$A \cap C \neq \emptyset$

c)





a) Find t_7.

t_7= t6+t5+t4

= 13+7+4=24

t6=t5+t4+t3

= 7+4+2= 13

t5=t4+t3+t2

=4+2+1=7

t4=t3+t2+t1

=2+1+1=4

t3=t2+t1+t0

= 1+1+0=2

b) Write a recursive algorithm to compute t_n, $n \geq 3$.

Input: n positive integer

Output: t(n)

t(n)

an = 3(an-1 + an-2), n > 2

7. If f :**R**→ **R** and g:**R**→ **R** are both one-to-one, is $f + g$ also one-to-one? Justify your answer.

if f x1 = f x2 ,then x1 = x2 Or, equivalently, if x1 ≠ x2,then f x1 ≠ f x2 .
Symbolically, f : X ®Y is one-to-one Û"x1 , x2 Î X, if f (x1) = f (x2) then x1 = x2 .
So yes it is one-to-one.

8. With each step you take when climbing a staircase, you can move up either one stair or two stairs. As a result, you can climb the entire staircase taking one stair at a time, taking two at a time, or taking a combination of one- or two-stair increments. For each integer n≥1, if the staircase consists of n stairs, let c_n be the number of different ways to climb the staircase. Find a recurrence relation for $c_1, c_2, \ldots, c_n$.

Given We can climb a staircase using 1 stair at a time or 2 stairs at a time

or any combination of l-stair and 2-stair steps

Cn=Number of different ways to climb a staircase with n stairs.

When n =

1, the staircase only contains 1 stair and thus we can only take

the staircase by using 1 stair at a time once, which is exactly 1 way

=1

When n = 2 the staircase only contains 2 stairs. We can then take the 2

stairs at one or take the stairs one by one, which thus results in 2 different

ways.

C2 = 2

When n > 3, the staircase contains more than 2 stairs and thus we will need

to use a combination of 1-stair and 2-stair steps

If the last move will be a 1-stair step, then there were an- 1 ways to arrive

at the previous stair (which was a staircase with n 1 stairs

If the last move will be a 2-stair step. then there were an-2 ways to arrive

at the previous stair (which was a staircase with n - 2 stairs)

The total number of ways is then the sum of the number of ways in which

the last move is a l-stair step and the number of ways in which the last

move is a 2-stair step.

C = 1 + C-2

C1 = 1, C2= 2, Cn = Cn-1 + Cn-2 when n >= 3

9. The Tribonacci sequence (t_n) is defined by the equations,

$t_0 = 0,\ t_1 = t_2 = 1, \quad t_n = t_{n-1} + t_{n-2} + t_{n-3}$ for all $n \geq 3$.

0,1,1,…..

(6) $\{1, 2, 3\}$

$$R_1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \qquad R_2 \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$R_1 = \{(1,1), (2,2), (2,3), (3,1), (3,3)\}$$

$$R_2 = \{(1,2), (2,2), (3,1), (3,3)\}$$

(a) $R_1 \cup R_2 = \{(1,1), (2,2), (2,3), (3,1), (3,3), (1,2)\}$

(b) $R_1 \cap R_2 = \{(2,2), (3,3), (3,1)\}$

$R_1 \cap R_2$ (b)

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ 3 & 1 & 0 & 1 \end{array}$$

(a) $R_1 \cup R_2$

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 1 & 1 & 0 \\ 2 & 0 & 1 & 1 \\ 3 & 1 & 0 & 1 \end{array}$$

(c)

① R_1 = Not Reflexive because $(4,4)$, $(5,5) \notin R_1$

② R_1 = Symetric $(1,2)$, $(2,1)$, $(1,3)$, $(3,1)$ $\in R_1$

③ R_1 = Is transitive

$(3,2)$ $(2,3)$ $(3,3)$ $\in R_1$

R_1 = . from ①,②,③ R_1 Not Equivalence

(d)

R_2 = Not reflexive $\left((1,1), (3,3), (5,5), (2,2), (4,4), \right) \notin R_2$

R_2 = Not Symmetric $(1,2) \notin R_2$ but $(2,1) \in R_2$

R_2 = trasitive because $(4,2)$, $(2,1)$, $(4,1)$ $\in R_2$

5

$$R = \{(x, y) \mid x + y \leq 6\}$$

$$R_1 = \{(y, z) \mid y \geq z\}$$

$$R_2 = y \rightarrow z$$

$$R_1 = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (4,1), (4,2), (5,1)\}$$

$$R_2 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3), (5,1), (5,2), (5,3), (5,4)\}$$

(b)

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

(a)

$$\begin{array}{c c} & \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array}$$

(4)

q	p	$\neg p$	$\neg q$	$\neg p \wedge q$	$\neg p \wedge \neg q$	$p \wedge q$	$\neg((\neg p \wedge q) \vee (\neg p \wedge \neg q))$	$(\neg p \wedge q \; \neg p \wedge \neg q)$	$\neg[(\neg p \wedge q) \vee (\neg p \wedge \neg q)] \vee (p \wedge q)$
T	T	F	F	F	F	T	T	F	T
T	F	T	F	T	F	F	F	T	F
F	T	F	T	F	F	F	T	F	T
F	F	T	T	F	T	F	F	T	T

$S \cap T = \{(x,y) \in A{\times}B \mid (x,y) \in S \text{ and } (x,y) \in \mathrm{T}\}$

$S \cup T = \{(x,y) \in A{\times}B \mid (x,y) \in S \text{ or } (x,y) \in \mathrm{T}\}$

Let $A=\{-1, 1, 2, 4\}$ and $B=\{1,2\}$ and defined binary relations S and T from A to B as follows:

For all $(x,y) \in A{\times}B, \quad x\, S\, y \leftrightarrow |x| = |y|$

For all $(x,y) \in A{\times}B, \quad x\, T\, y \leftrightarrow x-y$ is even

State explicitly which ordered pairs are in $A{\times}B$, S, T, $S \cap T$, and $S \cup T$.

AxB={ (-1,1),(-1,2),(1,1),(1,2),(2,1),(2,2),(4,1),(4,2)}

S={(-1,1),(1,1),(2,2)}

T={(-1,1),(1,1),(2,2),(4,2)}

S∩T={(-1,1),(1,1),(2,2)}

S∪T={(-1,1), (1,1),(2,2),(4,2)}

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$A' = U - A = \{3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

$B' = U - B = \{4, 5, 6, 7, 8, 9, 10, \ldots\}$

$\cancel{B'AA}\ A' \cap B' = \{4, 5, 6, 7, 8, 9, 10, \ldots\}$

c) $A' \cup B' = \{\cancel{1, 2,}\ 3, 4, 5, 6, 7, 8, 9, 10, \ldots\}$

Q_2.