

Q₁: Let $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$B = \{2, 5, 9\}$

$C = \{a, b\}$ find each of the following:-

i. $A - B$:

$$\begin{aligned} & \{1, 2, 3, 4, 5, 6, 7, 8\} - \{2, 5, 9\} \\ & = \{1, 3, 4, 6, 7, 8\} \end{aligned}$$

$$\begin{aligned} \text{(ii)} \quad (A \cap B) \cup C &= \{2, 5\} \cup \{a, b\} \\ &= \{2, 5, a, b\} \end{aligned}$$

$$\begin{aligned} \text{(iii)} \quad A \cap B \cap C &= \{2, 5\} \cap \{a, b\} \\ &= \emptyset, \text{ empty set} \end{aligned}$$

$$\begin{aligned} \text{(iv)} \quad B \times C &= \{2, 5, 9\} \times \{a, b\} \\ &= \{(2, a), (2, b), (5, a), (5, b), \\ & \quad (9, a), (9, b)\} \end{aligned}$$

$$\begin{aligned} \text{(v)} \quad P(C) &= \text{power set of } C \\ &= \text{set of all subset of } C \\ &= \{\emptyset, \{a\}, \{b\}, \{a, b\}\} \end{aligned}$$

Q. b) $(P \wedge ((P' \vee Q)')) \vee (P \wedge Q)$
 $= (P \wedge ((P')' \wedge Q)) \vee (P \wedge Q)$, By De Morgan's Law.
 $= P \wedge (P \vee B) \vee (P \wedge B)$
 $= ((P \wedge P) \wedge B) \vee (P \wedge B)$
 $= (P \wedge B) \vee (P \wedge B)$
 $= P \wedge (B' \vee B)$
 $= \emptyset$

c)

P	Q	$\neg P$	$\neg P \wedge Q$	$Q \leftrightarrow P$	A
T	T	F	F	T	T
T	F	F	F	F	\overline{T}
F	T	T	T	F	F
F	F	T	F	T	T



Q1) d) $x=3 \rightarrow (x+2)^2$

$P(x) = x$ is an odd

$Q(x) = x^2$ is an odd integer $(3+2)^2 = 25$ (both odd)

$\forall x (P(x) \rightarrow Q(x))$

Let a is odd integer

$a = 2n + 1$

~~$a^2 = (2n+1)^2$~~

$(a+2)^2 = (2n+1+2)^2$

$(a+2)^2 = (2n+3)^2$

~~$(a+2)^2 = 4n^2 + 12n + 9$~~

$(a+2)^2 = 4(n^2 + 3n) + 9$

$(a+2)^2 = 4m + 9$

$a+2^2 = 1$ is an odd integer

~~Q~~ Q₁)

e) i)

- Consider the positive integers $x=5, y=3$

$5 \geq 3 \Rightarrow x \geq y \Rightarrow P(x, y)$ is true.

\exists is the existential quantifier. Hence it is true that $\exists x \exists y P(x, y)$.

ii) Consider the positive integers $x=2, y=3$

$2 \not\geq 3 \Rightarrow x \not\geq y \Rightarrow P(x, y)$ is not true

\forall is the universal quantifier.

Hence, it is FALSE that $\forall x \forall y P(x, y)$.

Q2) a)

i) Therefore elements of relation R will be:

$$R = \{(4,5), (6,4), (6,5)\}$$

$$\text{Domain of } R = \{4, 6\} \quad R = \{4, 6\}$$

$$\text{Range of } R = \{4, 5\}$$

$$\text{ii) } R = \{(4,5), (6,4), (6,5)\}$$

we notice that $(4,4) \notin R$, $(5,5) \notin R$
 $(6,6) \notin R$

Therefore R is irreflexive

relation
Therefore R is antisymmetric

relation
Therefore Relation is irreflexive
and antisymmetric.

Q2) b)

i) elements of set S

as $4+5 \geq 9$ & $5+5 \geq 9$

so the elements of the set S
are $\{4, 5\}$

~~so the elements of the set~~

ii) $S = \{4, 5\}$

- as $4+4 < 9$ so S is not reflexive

- $4+5 \geq 9$ & $5+4 \geq 9$ so set S is

symmetric

- $4+5 \geq 9$ & $5+5 \geq 9$ so $5+5 \geq 9$

so set S is transitive.

as S is not reflexive it does not
contain any equivalence relation.

Q2 C. i. f is clearly one to one but it's not onto and it doesn't take the value 4.

ii. g is onto and it takes both values 1, 2. It's not one to one, $g(1) \neq g(2)$.

iii. h isn't one to one as $h(1) = h(2)$ and isn't onto, cause it doesn't take the value 2.

d. i. $x = 4y + 3$, $4y = x - 3$

~~$y = \frac{x-3}{4}$~~ $y = \frac{x-3}{4}$

ii. $\text{Dom} = \text{Ran}(m(x)) = 2(4x+3) - 4$
 $= 8x + 6 - 4 = 8x + 2$

Q3 a) i. $a_1 = 1$

$$a_2 = a_1 + 4 = 1 + 4 = 5$$

$$a_3 = a_2 + 2(3) = a_2 + 6 = 5 + 6 = 11$$

first 3 items = 1, 5, 11

ii.
$$\begin{cases} a = 1 \\ a_k = a_{k-1} + 2k, k \geq 2 \end{cases}$$

b) $r_1 = 7$, $r_2 = 2r_1$, $r_3 = 2^2 r_2$

$$r_k = 2^{k-1} r_1 = \text{final recurrence relation.}$$

c) Input = n
output = $S(n)$

$S(n) \{$
 $S(n-1)$
 return 5
 return $5 * S(n-1)$
}

$$S(4) = 5 \cdot 5 \cdot 5 \cdot 5 = 625$$

$$S(4) = 625$$

Trace $S(4)$

Q4

(a)

$3 \rightarrow B = \{3, 4, 5, 6, 7, 8, 9, A, B\} = 9$ possibilities

$5 \rightarrow F = \{5, 6, 7, 8, 9, A, B, C, D, E, F\} = 11$ possibilities

\square \square \square \square Four digits
9 16 16 11

$9 \times 16 \times 16 \times 11 =$ Hexadecimal number

(b)

~~digit~~ letters digit
 \square \square \square \square \square \square \square
A 26 26 26 10 10 0

$26 \times 26 \times 26 \times 10 \times 10 =$ license plates

(c)

COMPUTER
1 2 3 4 5 6 7 8 \rightarrow For on letter

first letter \rightarrow 8 ways \rightarrow ~~8 ways~~

second letter \rightarrow 7 ways

$$N(A_2) = 8 \times 7 = 56$$

third letter \rightarrow 6 ways

$$N(A_3) = 8 \times 7 \times 6 = 336$$

$$\text{total}(A) = 8 + 56 + 336 = 400 \text{ arrangement}$$

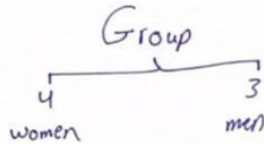
Q 4

(d)

total $\rightarrow 13$

women $\rightarrow 7$

men $\rightarrow 6$



women for group $\binom{7}{4} =$

men for group $\binom{6}{3} =$

total number for groups = $\binom{7}{4} \times \binom{6}{3}$

(e)

PROBABILITY

number of letters = 11

B Repeating times = 2

I Repeating times = 2

word PROBABILITY can be arranged = $\frac{11!}{2!2!} =$

(f)

$n = 6$

$r = 10$

$$C \binom{6+10-1}{10} = \binom{15}{10} = \frac{15!}{10!} =$$

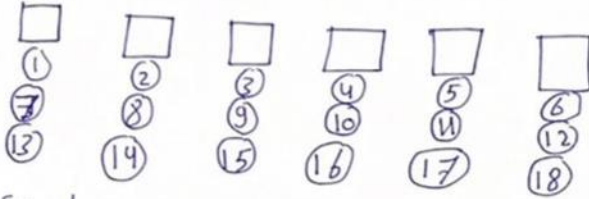
Q5

(a)

The person's first and last name are

$3 \times 2 = 6 \Rightarrow$ pegionholes \Rightarrow probabilities of the name

$18 \Rightarrow$ pegion \Rightarrow person's



So by pegionhole principle some of pegionholes have at least three pegions.

(b)

even numbers = $\{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\} = 10$ numbers

odd numbers = $\{1, 3, 5, 7, 9, 11, 13, 15, 17, 19\} = 10$ numbers

~~10 + 10 = 20 numbers~~

$10 + 1 = 11$ integer numbers

(c)

$$\begin{aligned} a &= 5 \\ d &= 5 \\ n &=? \\ a_n &= 100 \end{aligned}$$

$$\begin{aligned} a_n &= 10 \\ a + (n-1)d &= 100 \\ 5 + (n-1)5 &= 100 \end{aligned}$$

$$(n-1)5 = 100 - 5$$

$$\frac{(n-1)5}{5} = \frac{95}{5}$$

$$n-1 = 19$$

$$n = 20 \text{ integers}$$

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