UNIVERSITI TEKNOLOGI MALAYSIA

TUTORIAL 3

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SECTION : SECI1013 – 09

Question 1

a) i. $A - B = \{1, 3, 4, 6, 7, 8\}$

ii. $(A \cap B) \cup C$

$A \cap B = \{2, 5\}$

$(A \cap B) \cup C = \{2, 5, a, b\}$

iii. $A \cap B \cap C = \{\ \}$

iv. $B \times C = \{(2,a), (2,b), (5,a), (5,b), (9,a), (9,b)\}$

v. $P(C) = 2^2$

$= 4$

$= \{\{\}, \{a\}, \{b\}, \{a,b\}\}$

b) $(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$

$(P \cap ((P'' \cap Q'))) \cup (P \cap Q)$ (De Morgan's Thereom)

$P \cap ((P \cap Q'))) \cup (P \cap Q)$ (complement law)

$(P \cap P \cap Q') \cup (P \cap Q)$

$(P \cap Q') \cup (P \cap Q)$ (Idempotent laws)

$P \cap Q' \cup (P \cap Q)$

$P \cap (Q \cup P) \cap (Q' \cup Q)$ (Distributive laws)

$P \cap (Q \cup P) \cap U$ (Complement laws)

$P \cap U$ (Absorption laws)

P (Properties of universal set)

c)

P	q	¬p	¬p ∨ q	q → p	A
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) - if $x = 3$, then $(x+2)^2 = 25$ so $(x+2)^2$ is odd.

- if $x = 5$, then $(x+2)^2 = 49$ so $(x+2)^2$ is odd.

let $P(x) = x$ is odd

$S(x) = (x+2)^2$ is odd

$\forall x\ (P(x) \rightarrow S(x))$

Let a is an odd integer,

$a = 2n+1$

$(a+2)^2 = (2n+1+2)^2$

$= (2n+3)^2$

$= 4n^2 + 12n + 9$

$= 2(2n^2 + 6n) + 9$

$= 2m + 9$ where m is an integer

$(a+2)^2$ is an odd integer

$\therefore$ for all integer x, if x is odd, then $(x+2)^2$ is odd.

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e) i. $\exists x \exists y \cdot P(x, y)$

Let $x = 2$, $y = 1$ then $x \geq y$

Let $x = 1$, $y = 1$ then $x \geq y$

Let $x = 3$, $y = 4$ then $x \leq y$

$\therefore$ The statement is TRUE

ii. $\forall x \forall y \ P(x, y)$

Let $x = 1$, $y = 2$ then $x \leq y$

Let $x = 2$, $y = 1$ then $x \geq y$

$\therefore$ The statement is FALSE

Question 2

a) $R = \{(1,1),(1,2),(2,2),(1,3)\}$

i. Domain = $\{1,2\}$

Range = $\{1,2,3\}$

ii. - R is not irreflexive since there is no diagonal 0 in the matrix.

- R is antisymmetric since

$(1,1) \in R$ and $(1,1) \in R$ implies $a=b$

$(2,2) \in R$ and $(2,2) \in R$ implies $a=b$

$(1,2) \in R$ but $(2,1) \notin R$

$(1,3) \in R$ but $(3,1) \notin R$

b) i. $S = \{(4,5),(5,4),(5,5)\}$

ii. $M_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

- S is not reflexive since there is no diagonal 1 in M_S.

$M_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ $M_S^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

- S is symmetric since $M_S = M_S^T$

$M_S \otimes M_S =$

$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

- S is transitive since $M_S \otimes M_S = M_S$

- S is not equivalence relation since S is not reflexive.

c) i. $f: X \to Y = \{(1,2),(2,4),(3,1)\}$

ii. $g: X \to Z = \{(1,1),(1,2),(2,2),(2,1),(3,2)\}$

iii. $h: X \to X = \{(1,1),(1,2),(2,1),(2,2),(3,1)\}$

d) i) $m(x) = 4x+3$

$m^{-1}(y) = 4x+3$

$y = 4x+3$

$y-3 = 4x$

$x = \frac{y-3}{4}$

$m^{-1}(y) = \frac{y-3}{4}$

$m^{-1}(x) = \frac{x-3}{4}$

ii) $n \circ m = n(m(x))$

$= 2(4x+3)-4$

$= 8x+6-4$

$= 8x-2$

Question 3

a) i) $a_1 = 1$

$a_2 = a_{(2-1)} + 2(2)$
$= a_1 + 4$
$= 1 + 4$
$= 5$

$a_3 = a_{(3-1)} + 2(3)$
$= a_2 + 6$
$= 5 + 6$
$= 11$

$\therefore$ The first three terms are 1, 5, 11

ii) Input : n
Output : f(n)

```
f(n) {
    if (n=1)
        return 1
    return f(n-1) + 2*n
}
```

b) $r_k = 2r_{k-1}$, $k > 1$, where $r_1 = 7$

c) when $n = 1$
because $n = 1$
return 5
$\therefore S(1) = 5$

when $n = 2$
because $n \neq 1$
return $5 * S(2-1)$
$\therefore S(2) = 5 * S(1)$
$= 5 * 5$
$= 25$

when $n = 3$
because $n \neq 1$
return $5 * S(3-1)$
$\therefore S(3) = 5 * S(2)$
$= 5 * 25$
$= 125$

when $n = 4$
because $n \neq 1$
return $5 * S(4-1)$
$\therefore S(4) = 5 * S(3)$
$= 5 * 125$
$= 625$

$\therefore S(4) = 625$

Question 4

a) Total ways to create a 4 digits long hexadecimal number begins with one of the digits 3 through B, ends with one of the digits 5 through F:

$\underline{9} \times \underline{16} \times \underline{16} \times \underline{11} = 25344$ ways

b) Total ways to create an automobile license plate begins with A and ends in 0:

$\underline{1} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{1} = 1757600$ ways

c) For three letters row:
Number of ways $= 8 \times 7 \times 6 = 336$ ways

For two letters row:
Number of ways $= 8 \times 7 = 56$ ways

For one letter row:
Number of ways $= 8$ ways

Total number of ways to arrange COMPUTER in a row of no more than three letters with no repetitions $= 336 + 56 + 8$
$= 400$ ways

d) Number of ways to choose a group of seven that contains 4 women and 3 men:
${}^{7}C_{4} \times {}^{6}C_{3} = 700$ ways

e) Total number of characters in PROBABILITY = 11
Characters with 2 repetitions = B and I
Number of distinguishable ways to arrange PROBABILITY $= \dfrac{11!}{2!\,2!}$
$= 9979200$ ways

f) Number of selections of ten pastries $= \dfrac{(6+10-1)!}{10!\,(6-1)!}$
$= 3003$ ways

Question 5

First Name

a) 18 persons (pigeons)

Ali
Bahar
Carlie $\}$ 3

Last Name

David
Elyas $\}$ 2

pigeonholes

$x \geq 3$

x = people hv same last & first name

$n = 18$

$k = 3 \times 2 = 6$

$x = n/k$

$= 18/6$

$= 3$ people

$\therefore$ $x = 3$, $x \geq 3$

there are at least 3 ppl with same first & last name by using generalized pigeonhole principle.

b) $1 \xrightarrow{x \text{ odd}} 20$

$x \geq 1$

odd intergers = 1, 3, 5, ... 19

= 10 in total

even intergers = 2, 4, 6, ... 20

= 10 in total

$n = 10$

$x = n + 1$

$= 10 + 1$

$= 11$ intergers

c) $1 \xrightarrow{x \to /5 \checkmark} 100$

intergers that are divisable by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

= 20 in total

intergers that is not divisable by 5 = 100 − 20

= 80 ← k

$x = k + 1$

$= 80 + 1$

$= 81$

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