



## TUTORIAL 3

**GROUP NAME : T5**

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**SECTION : SECI1013 – 09**

# Question 1

i.  $A - B = \{1, 3, 4, 6, 7, 8\}$

ii.  $(A \cap B) \cup C$

$A \cap B = \{2, 5\}$

$(A \cap B) \cup C = \{2, 5, a, b\}$

iii.  $A \cap B \cap C = \{ \}$

iv.  $B \times C = \{(2, a), (2, b), (5, a), (5, b), (9, a), (9, b)\}$

v.  $P(C) = 2^2$

$= 4$

$= \{ \{ \}, \{a\}, \{b\}, \{a, b\} \}$

b)  $(P \cap ((P' \cup Q)')) \cup (P \cap Q) = P$

$(P \cap ((P' \cap Q)')) \cup (P \cap Q)$

(De Morgan's Theorem)

$P \cap ((P \cap Q)') \cup (P \cap Q)$

(complement law)

$(P \cap P \cap Q) \cup (P \cap Q)$

(Idempotent laws)

$(P \cap Q) \cup (P \cap Q)$

$P \cap Q \cup (P \cap Q)$

$P \cap (Q \cup P) \cap (Q' \cup Q)$

(Distributive laws)

$P \cap (Q \cup P) \cap U$

(Complement laws)

$P \cap U$

(Absorption laws)

$P$

(Properties of universal set)

c)

P	Q	$\neg P$	$\neg P \vee Q$	$Q \rightarrow P$	A
T	T	F	T	T	T
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	T	T	T

d) - if  $x = 3$ , then  $(x+2)^2 = 25$  so  $(x+2)^2$  is odd.

- if  $x = 5$ , then  $(x+2)^2 = 49$  so  $(x+2)^2$  is odd.

let  $P(x) = x$  is odd

$S(x) = (x+2)^2$  is odd

$\forall x (P(x) \rightarrow S(x))$

Let  $a$  is an odd integer,

$a = 2n + 1$

$(a+2)^2 = (2n+1+2)^2$

$= (2n+3)^2$

$= 4n^2 + 12n + 9$

$= 2(2n^2 + 6n) + 9$

$= 2m + 9$  where  $m$  is an integer

$(a+2)^2$  is an odd integer

$\therefore$  for all integer  $x$ , if  $x$  is odd, then  $(x+2)^2$  is odd.

e) i.  $\exists x \exists y \cdot P(x, y)$

Let  $x = 2, y = 1$  then  $x \geq y$

Let  $x = 1, y = 1$  then  $x \geq y$

Let  $x = 3, y = 4$  then  $x \leq y$

$\therefore$  The statement is TRUE

ii.  $\forall x \forall y P(x, y)$

Let  $x = 1, y = 2$  then  $x \leq y$

Let  $x = 2, y = 1$  then  $x \geq y$

$\therefore$  The statement is FALSE

Question 2

a)  $R = \{(1,1), (1,2), (2,2), (1,3)\}$

i. Domain =  $\{1,2\}$

Range =  $\{1,2,3\}$

ii. - R is not reflexive since there is no diagonal 0 in the matrix.

- R is antisymmetric since

$(1,1) \in R$  and  $(1,1) \in R$  implies  $a=b$

$(2,2) \in R$  and  $(2,2) \in R$  implies  $a=b$

$(1,2) \in R$  but  $(2,1) \notin R$

$(1,3) \in R$  but  $(3,1) \notin R$

b) i.  $S = \{(4,5), (5,4), (5,5)\}$

ii.  $M_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

- S is not reflexive since there is no diagonal 1 in  $M_S$ .

$M_S = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$       $M_S^T = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

- S is symmetric since  $M_S = M_S^T$

$M_S \otimes M_S =$

$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

- S is transitive since  $M_S \otimes M_S = M_S$

- S is not equivalence relation since S is not reflexive.

c) i.  $f: X \rightarrow Y = \{(1,2), (2,4), (3,1)\}$

ii.  $g: X \rightarrow Z = \{(1,1), (1,2), (2,2), (2,1), (3,2)\}$

iii.  $h: X \rightarrow X = \{(1,1), (1,2), (2,1), (2,2), (3,1)\}$

$$\begin{aligned} \text{d) i) } m(x) &= 4x+3 \\ m^{-1}(y) &= 4x+3 \\ y &= 4x+3 \\ y-3 &= 4x \\ x &= \frac{y-3}{4} \\ m^{-1}(y) &= \frac{y-3}{4} \\ m^{-1}(x) &= \frac{x-3}{4} \end{aligned}$$

$$\begin{aligned} \text{ii) } nom &= n(m(x)) \\ &= 2(4x+3)-4 \\ &= 8x+6-4 \\ &= 8x-2 \end{aligned}$$

Question 3

a) i)  $a_1 = 1$

$$\begin{aligned} a_2 &= a_{(2-1)} + 2(2) \\ &= a_1 + 4 \\ &= 1 + 4 \\ &= 5 \end{aligned}$$

$$\begin{aligned} a_3 &= a_{(3-1)} + 2(3) \\ &= a_2 + 6 \\ &= 5 + 6 \\ &= 11 \end{aligned}$$

$\therefore$  The first three terms are 1, 5, 11

ii) Input: n

Output:  $f(n)$

```
f(n) {
    if (n=1)
        return 1
    return f(n-1) + 2*n
}
```

b)  $r_k = 2r_{k-1}$ ,  $k > 1$ , where  $r_1 = 7$

c) when  $n = 1$

because  $n = 1$

return 5

$$\therefore S(1) = 5$$

when  $n = 2$

because  $n \neq 1$

return  $5 * S(2-1)$

$$\therefore S(2) = 5 * S(1)$$

$$= 5 * 5$$

$$= 25$$

when  $n = 3$

because  $n \neq 1$

return  $5 * S(3-1)$

$$\therefore S(3) = 5 * S(2)$$

$$= 5 * 25$$

$$= 125$$

when  $n = 4$

because  $n \neq 1$

return  $5 * S(4-1)$

$$\therefore S(4) = 5 * S(3)$$

$$= 5 * 125$$

$$= 625$$

$$\therefore S(4) = 625$$

Question 4

- a) Total ways to create a 4 digits long hexadecimal number begins with one of the digits 3 through B, ends with one of the digits 5 through F:

$$\underline{9} \times \underline{16} \times \underline{16} \times \underline{11} = 25344 \text{ ways}$$

- b) Total ways to create an automobile license plate begins with A and ends in O:

$$\underline{1} \times \underline{26} \times \underline{26} \times \underline{26} \times \underline{10} \times \underline{10} \times \underline{1} = 1757600 \text{ ways}$$

- c) For three letters row:

$$\text{Number of ways} = 8 \times 7 \times 6 = 336 \text{ ways}$$

For two letters row:

$$\text{Number of ways} = 8 \times 7 = 56 \text{ ways}$$

For one letter row:

$$\text{Number of ways} = 8 \text{ ways}$$

Total number of ways to arrange COMPUTER in a row of no more than three letters with no repetitions =  $336 + 56 + 8$   
= 400 ways

- d) Number of ways to choose a group of seven that contains 4 women and 3 men:

$${}^7C_4 \times {}^6C_3 = 700 \text{ ways}$$

- e) Total number of characters in PROBABILITY = 11

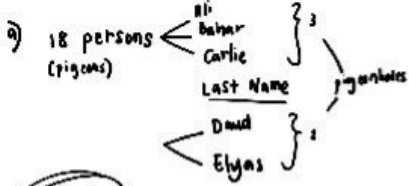
Characters with 2 repetitions = B and I

$$\text{Number of distinguishable ways to arrange PROBABILITY} = \frac{11!}{2!2!} = 9979200 \text{ ways}$$

- f) Number of selections of ten pastries =  $\frac{(6+10-1)!}{10!(6-1)!}$

$$= 3003 \text{ ways}$$

Question 5



$x \geq 3$

$x$  = people hv same last & first name

$n = 18$

$k = 3 \times 2 = 6$

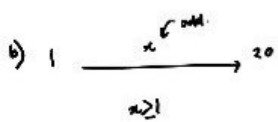
$x = n/k$

$= 18/6$

$= 3$  people

$\therefore x = 3, x \geq 3$

there are at least 3 ppl with same first & last name by using generalized pigeonhole principle.



odd integers = 1, 3, 5, ... 19

$= 10$  in total

even integers = 2, 4, 6, ... 20

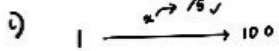
$= 10$  in total

$n = 10$

$x = n + 1$

$= 10 + 1$

$= 11$  integers



integers that are divisible by 5 = 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100

$= 20$  in total

integers that is not divisible by 5 = 100 - 20  
 $= 80$   $\leftarrow k$

$x = k + 1$

$= 80 + 1$

$= 81$