Chapter 5, Problem 1.

The equivalent model of a certain op amp is shown in Fig. 5.43. Determine:
(a) the input resistance.
(b) the output resistance.
(c) the voltage gain in dB.

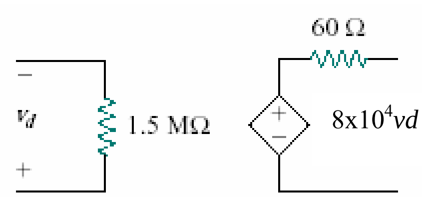


Figure 5.42 For Prob. 5.1.

Chapter 5, Solution 1.

(a) R_{in} = **<u>1.5 MΩ</u>**
(b) R_{out} = **<u>60 Ω</u>**
(c) $A = 8x10^4$
Therefore $A_{dB} = 20 \log 8x10^4$ = **<u>98.0 dB</u>**

Chapter 5, Problem 2

The open-loop gain of an op amp is 100,000. Calculate the output voltage when there are inputs of +10 μV on the inverting terminal and + 20 μV on the noninverting terminal.

Chapter 5, Solution 2.

$$v_0 = Av_d = A(v_2 - v_1)$$

$$= 10^5 (20\text{-}10) \times 10^{-6} = \underline{\mathbf{1V}}$$

Chapter 5, Problem 3

Determine the output voltage when .20 μV is applied to the inverting terminal of an op amp and +30 μV to its noninverting terminal. Assume that the op amp has an open-loop gain of 200,000.

Chapter 5, Solution 3.

$$v_0 = Av_d = A(v_2 - v_1)$$
$$= 2 \times 10^5 (30 + 20) \times 10^{-6} = \underline{\mathbf{10V}}$$

Chapter 5, Problem 4

The output voltage of an op amp is .4 V when the noninverting input is 1 mV. If the open-loop gain of the op amp is 2×10^6, what is the inverting input?

Chapter 5, Solution 4.

$$v_0 = Av_d = A(v_2 - v_1)$$
$$v_2 - v_1 = \frac{v_0}{A} = \frac{-4}{2x10^6} = -2\mu V$$

$$v_2 - v_1 = -2\ \mu V = -0.002\ mV$$
$$1\ mV - v_1 = -0.002\ mV$$
$$v_1 = \underline{\mathbf{1.002\ mV}}$$

Chapter 5, Problem 5.

For the op amp circuit of Fig. 5.44, the op amp has an open-loop gain of 100,000, an input resistance of 10 kΩ, and an output resistance of 100 Ω. Find the voltage gain v_o/v_i using the nonideal model of the op amp.

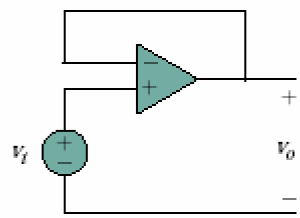
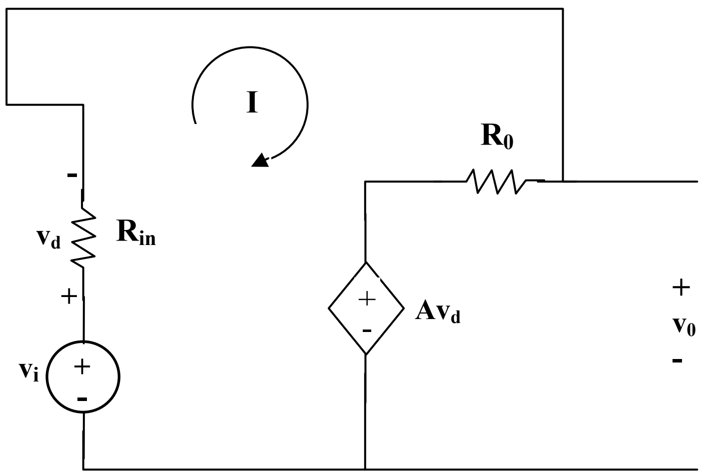


Figure 5.44 for Prob. 5.5

Chapter 5, Solution 5.



$$-v_i + Av_d + (R_i + R_0)\, I = 0 \qquad (1)$$

But $\quad v_d = R_i I,$

$$-v_i + (R_i + R_0 + R_i A)\, I = 0$$

$$I = \frac{v_i}{R_0 + (1 + A)R_i} \qquad (2)$$

$$-Av_d - R_0 I + v_0 = 0$$

$$v_0 = Av_d + R_0 I = (R_0 + R_i A)I \;=\; \frac{(R_0 + R_i A)v_i}{R_0 + (1 + A)R_i}$$

$$\frac{v_0}{v_i} = \frac{R_0 + R_i A}{R_0 + (1 + A)R_i} = \frac{100 + 10^4 x 10^5}{100 + (1 + 10^5)} \cdot 10^4$$

$$\cong \frac{10^9}{\left(1 + 10^5\right)} \cdot 10^4 = \frac{100{,}000}{100{,}001} = \underline{\mathbf{0.9999990}}$$

Chapter 5, Problem 6

Using the same parameters for the 741 op amp in Example 5.1, find v_o in the op amp circuit of Fig. 5.45.

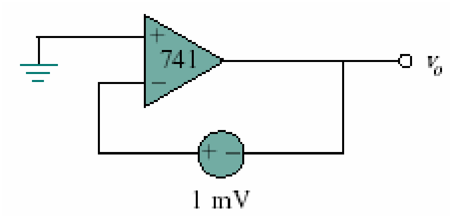
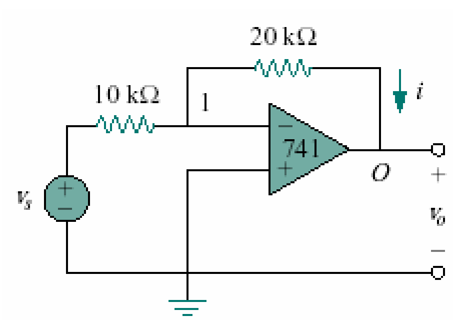


Figure 5.45 for Prob. 5.6

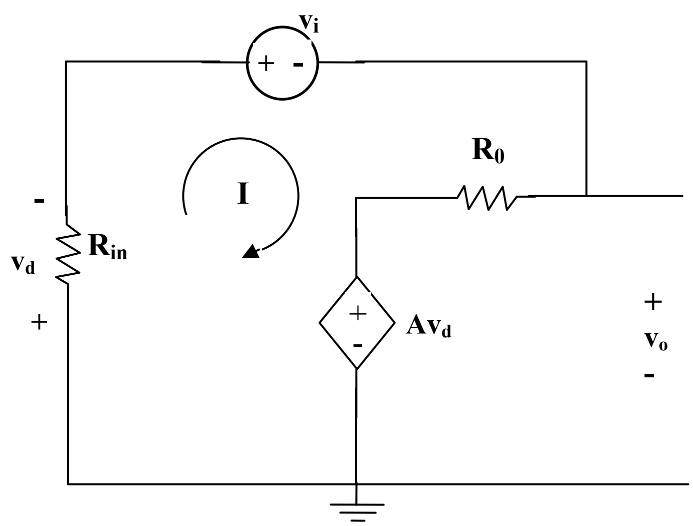
Example 5.1

A 741 op amp has an open-loop voltage gain of 2×10^5, input resistance of 2 MΩ, and output resistance of 50Ω. The op amp is used in the circuit of Fig. 5.6(a). Find the closed-loop gain v_o/v_s . Determine current i when v_s = 2 V.



(a)

Chapter 5, Solution 6.



$$(R_0 + R_i)R + v_i + Av_d = 0$$

But $\quad v_d = R_iI,$

$$v_i + (R_0 + R_i + R_iA)I = 0$$

$$I = \frac{-v_i}{R_0 + (1+A)R_i} \qquad (1)$$

$$-Av_d - R_0I + v_o = 0$$

$$v_o = Av_d + R_0I = (R_0 + R_iA)I$$

Substituting for I in (1),

$$v_0 = -\left(\frac{R_0 + R_iA}{R_0 + (1+A)R_i}\right)v_i$$

$$= -\frac{\left(50 + 2x10^6 x2x10^5\right)\cdot 10^{-3}}{50 + \left(1 + 2x10^5\right)x2x10^6}$$

$$\cong \frac{-200{,}000x2x10^6}{200{,}001x2x10^6}\text{mV}$$

$v_0 =$ **-0.999995 mV**

Chapter 5, Problem 7

The op amp in Fig. 5.46 has $R_i = 100\ \text{k}\Omega$, $R_o = 100\ \Omega$, $A = 100{,}000$. Find the differential voltage v_d and the output voltage v_o.

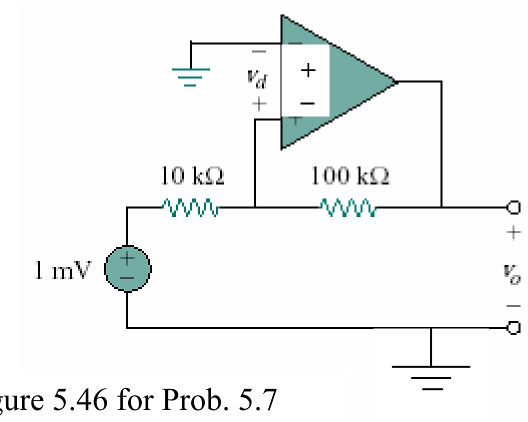
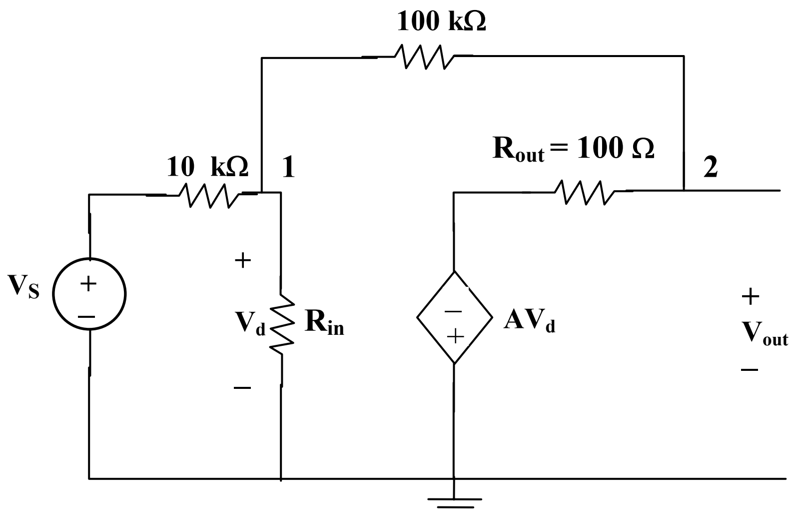


Figure 5.46 for Prob. 5.7

Chapter 5, Solution 7.



At node 1, $(V_S - V_1)/10\text{ k} = [V_1/100\text{ k}] + [(V_1 - V_0)/100\text{ k}]$

$$10\, V_S - 10\, V_1 = V_1 + V_1 - V_0$$

which leads to $V_1 = (10V_S + V_0)/12$

At node 2, $(V_1 - V_0)/100\text{ k} = (V_0 - (-AV_d))/100$

But $V_d = V_1$ and $A = 100{,}000$,

$$V_1 - V_0 = 1000\,(V_0 + 100{,}000V_1)$$

$$0 = 1001V_0 + 99{,}999{,}999[(10V_S + V_0)/12]$$

$$0 = 83{,}333{,}332.5\, V_S + 8{,}334{,}334.25\, V_0$$

which gives us $(V_0/ V_S) = -10$ (for all practical purposes)

If $V_S = 1$ mV, then $V_0 =$ **-10 mV**

Since $V_0 = A\, V_d = 100{,}000\, V_d$, then $V_d = (V_0/10^5)$ V $=$ **-100 nV**

Chapter 5, Problem 8

Obtain v_o for each of the op amp circuits in Fig. 5.47.

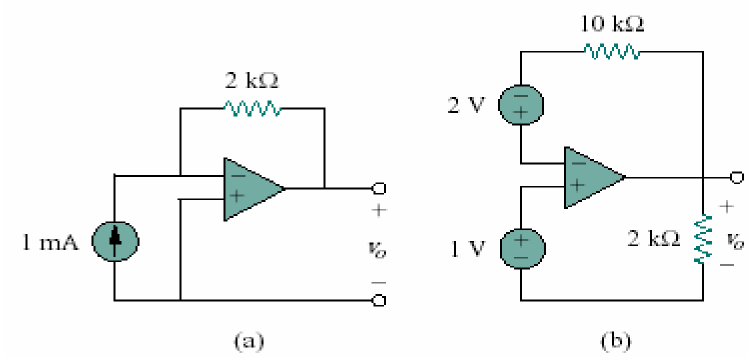


Figure 5.47 for Prob. 5.8

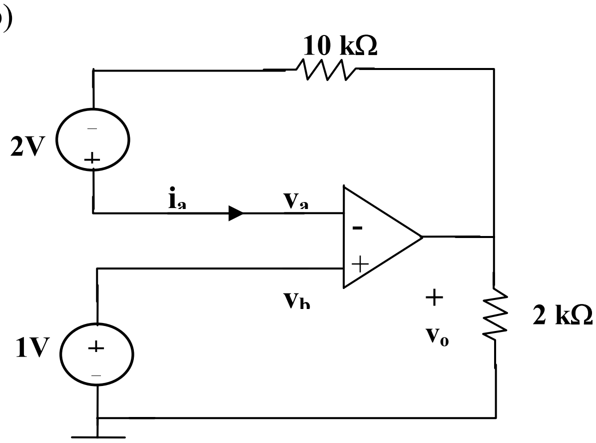
Chapter 5, Solution 8.

(a) If v_a and v_b are the voltages at the inverting and noninverting terminals of the op amp.

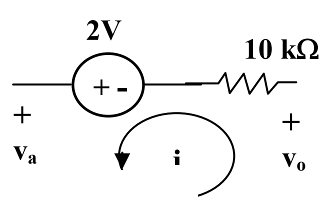
$v_a = v_b = 0$

$$1\text{mA} = \frac{0 - v_0}{2\text{k}} \longrightarrow v_0 = \underline{\mathbf{-2V}}$$

(b)



(a)



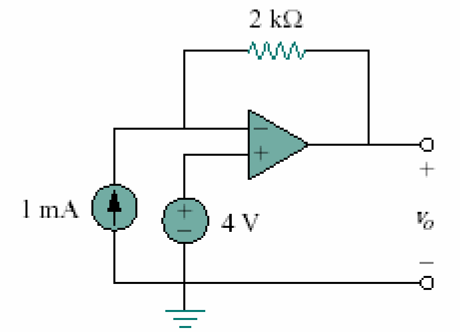
(b)

Since $v_a = v_b = 1$V and $i_a = 0$, no current flows through the 10 kΩ resistor. From Fig. (b),

$$-v_a + 2 + v_0 = 0 \longrightarrow v_0 = v_a - 2 = 1 - 2 = \underline{\mathbf{-1V}}$$

Chapter 5, Problem 9

Determine v_o for each of the op amp circuits in Fig. 5.48.



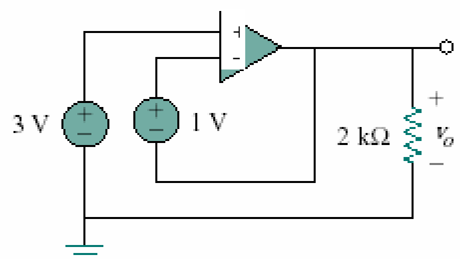


Figure 5.48 for Prob. 5.9

Chapter 5, Solution 9.

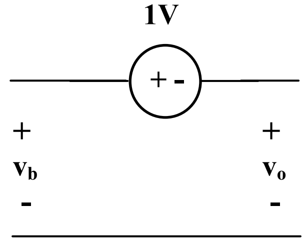
(a) Let v_a and v_b be respectively the voltages at the inverting and noninverting terminals of the op amp

$$v_a = v_b = 4V$$

At the inverting terminal,

$$1mA = \frac{4 - v_0}{2k} \longrightarrow v_0 = \underline{\mathbf{2V}}$$

(b)



Since $v_a = v_b = 3V$,

$$-v_b + 1 + v_o = 0 \longrightarrow v_o = v_b - 1 = \underline{\mathbf{2V}}$$

Chapter 5, Problem 10

Find the gain v_o/v_s of the circuit in Fig. 5.49.

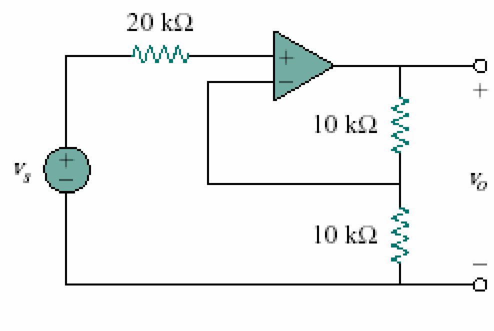


Figure 5.49 for Prob. 5.10

Chapter 5, Solution 10.

Since no current enters the op amp, the voltage at the input of the op amp is v_s. Hence

$$v_s = v_o\left(\frac{10}{10+10}\right) = \frac{v_o}{2} \longrightarrow \frac{v_o}{v_s} = \underline{\mathbf{2}}$$

Chapter 5, Problem 11

Find v_o and i_o in the circuit in Fig. 5.50.

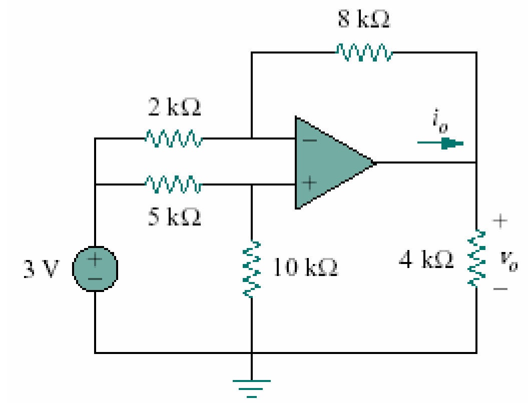
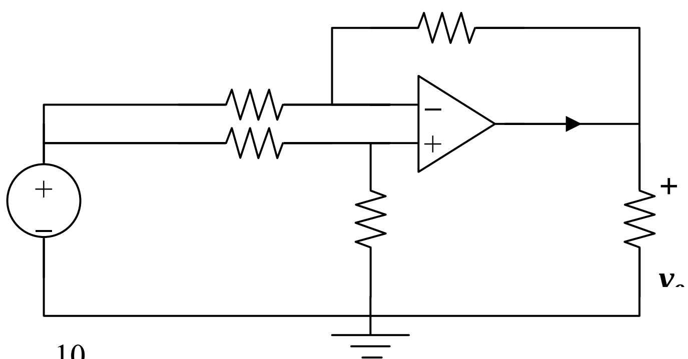


Figure 5.50 for Prob. 5.11

Chapter 5, Solution 11.



$$v_b = \frac{10}{10+5}(3) = 2V$$

At node a,

$$\frac{3 - v_a}{2} = \frac{v_a - v_o}{8} \longrightarrow 12 = 5v_a - v_o$$

But $v_a = v_b = 2V$,

$$12 = 10 - v_o \longrightarrow v_o = \underline{\mathbf{-2V}}$$

$$-i_o = \frac{v_a - v_o}{8} + \frac{0 - v_o}{4} = \frac{2+2}{8} + \frac{2}{4} = 1mA$$

$$i_o = \underline{\mathbf{-1mA}}$$

Chapter 5, Problem 12.

Calculate the voltage ratio v_o/v_s for the op amp circuit of Fig. 5.51. Assume that the op amp is ideal.

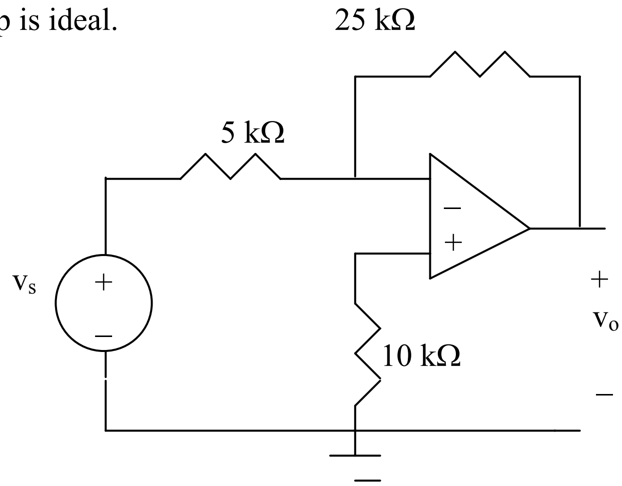


Figure 5.51 For Prob. 5.12.

Chapter 5, Solution 12.

This is an inverting amplifier.

$$v_o = -\frac{25}{5} v_s \quad \longrightarrow \quad \frac{v_o}{v_s} = \underline{-5}$$

Chapter 5, Problem 13

Find v_o and i_o in the circuit of Fig. 5.52.

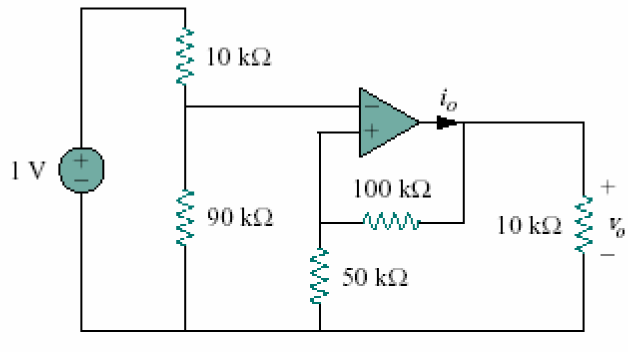
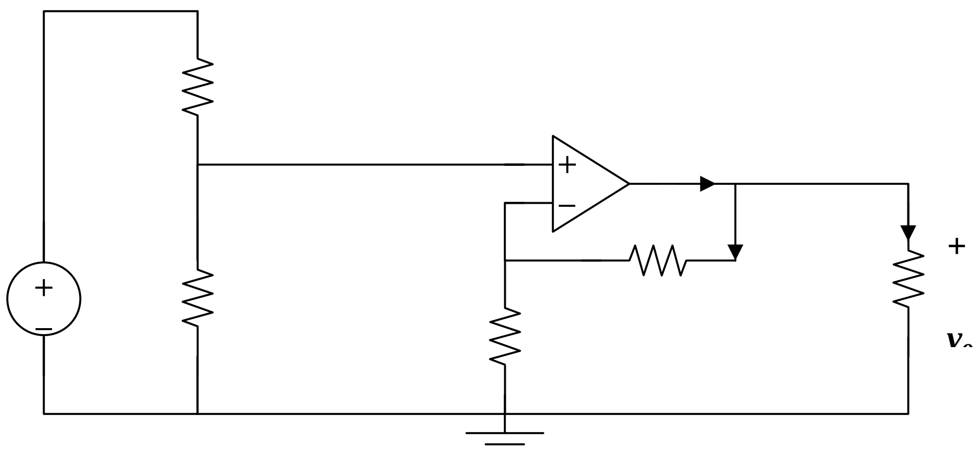


Figure 5.52 for Prob. 5.13

Chapter 5, Solution 13.



By voltage division,

$$v_a = \frac{90}{100}(1) = 0.9\text{V}$$

$$v_b = \frac{50}{150}v_o = \frac{v_o}{3}$$

But $v_a = v_b \longrightarrow \frac{v_0}{3} = 0.9 \longrightarrow v_o = \underline{\mathbf{2.7V}}$

$$i_o = i_1 + i_2 = \frac{v_o}{10k} + \frac{v_o}{150k} = 0.27\text{mA} + 0.018\text{mA} = \underline{\mathbf{288\ \mu A}}$$

Chapter 5, Problem 14

Determine the output voltage v_o in the circuit of Fig. 5.53.

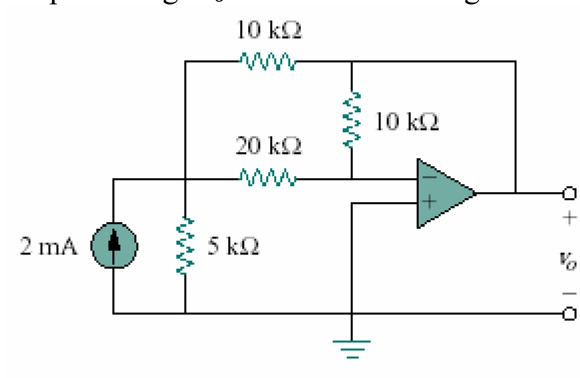
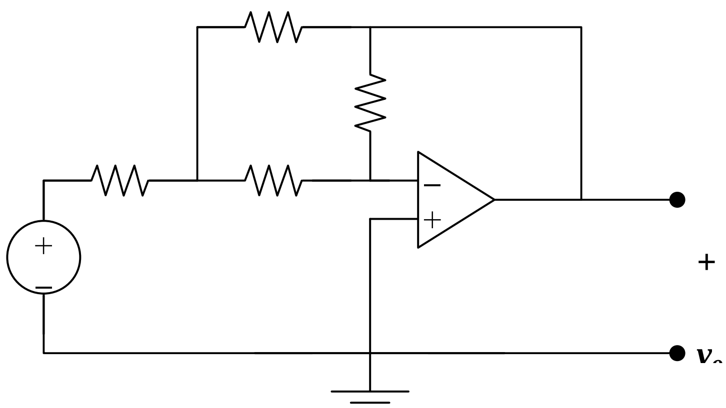


Figure 5.53 for Prob. 5.14

Chapter 5, Solution 14.

Transform the current source as shown below. At node 1,

$$\frac{10 - v_1}{5} = \frac{v_1 - v_2}{20} + \frac{v_1 - v_o}{10}$$



But $v_2 = 0$. Hence $40 - 4v_1 = v_1 + 2v_1 - 2v_o \longrightarrow 40 = 7v_1 - 2v_o$ (1)

At node 2, $\frac{v_1 - v_2}{20} = \frac{v_2 - v_o}{10}$, $v_2 = 0$ or $v_1 = -2v_o$ (2)

From (1) and (2), $40 = -14v_o - 2v_o \longrightarrow v_o =$ **-2.5V**

Chapter 5, Problem 15

(a). Determine the ratio v_o/i_s in the op amp circuit of Fig. 5.54.
(b). Evaluate the ratio for R_1 = 20 kΩ, R_2 = 25 kΩ, R_3 = 40 2kOmega$.

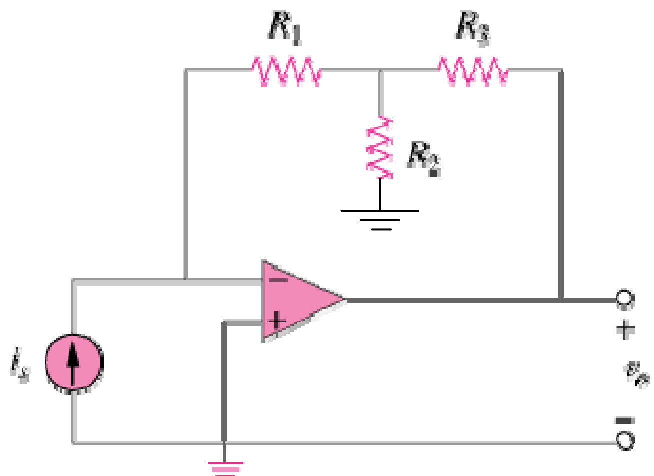


Figure 5.54

Chapter 5, Solution 15

(a) Let v_1 be the voltage at the node where the three resistors meet. Applying KCL at this node gives

$$i_s = \frac{v_1}{R_2} + \frac{v_1 - v_o}{R_3} = v_1\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{v_o}{R_3} \qquad (1)$$

At the inverting terminal,

$$i_s = \frac{0 - v_1}{R_1} \quad \longrightarrow \quad v_1 = -i_s R_1 \qquad (2)$$

Combining (1) and (2) leads to

$$i_s\left(1 + \frac{R_1}{R_2} + \frac{R_1}{R_3}\right) = -\frac{v_o}{R_3} \quad \longrightarrow \quad \underline{\frac{v_o}{i_s} = -\left(R_1 + R_3 + \frac{R_1 R_3}{R_2}\right)}$$

(b) For this case,

$$\frac{v_o}{i_s} = -\left(20 + 40 + \frac{20x40}{25}\right)\text{k}\Omega = \underline{\text{-}\,92\ \text{k}\Omega}$$

Chapter 5, Problem 16

Obtain i_x and i_y in the op amp circuit in Fig. 5.55.

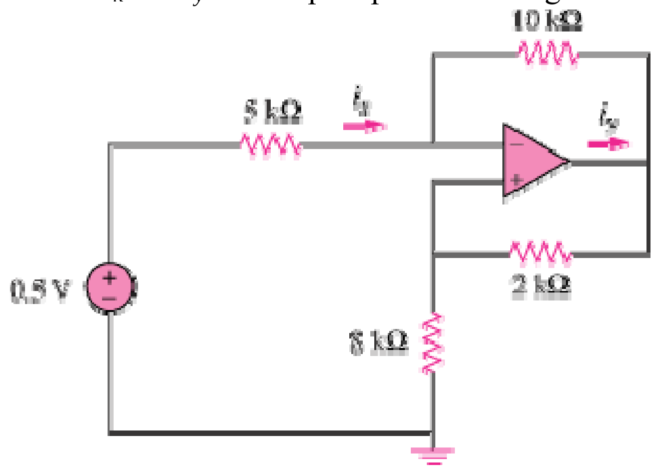
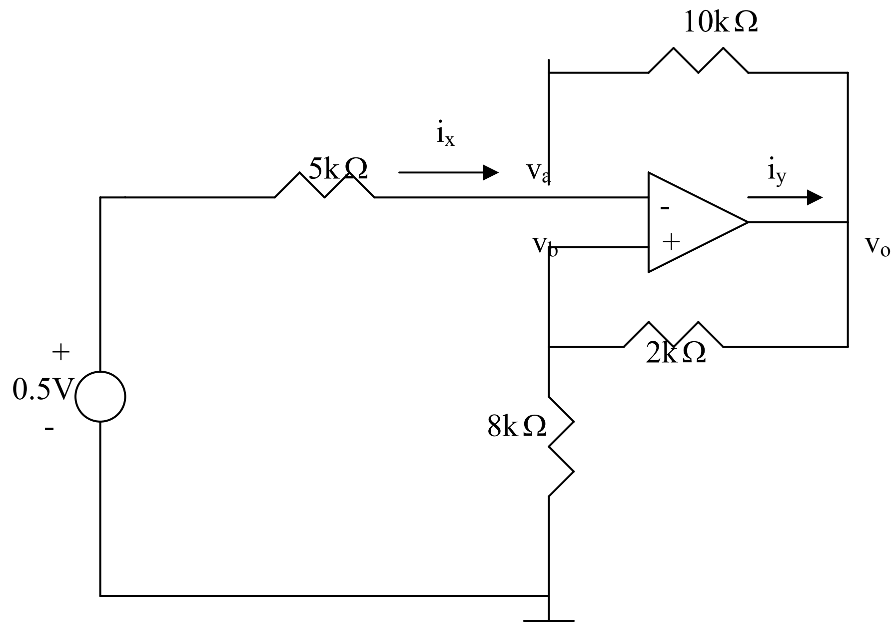


Figure 5.55

Chapter 5, Solution 16



Let currents be in mA and resistances be in kΩ. At node a,

$$\frac{0.5 - v_a}{5} = \frac{v_a - v_o}{10} \longrightarrow 1 = 3v_a - v_o \qquad (1)$$

But

$$v_a = v_b = \frac{8}{8+2}v_o \longrightarrow v_o = \frac{10}{8}v_a \qquad (2)$$

Substituting (2) into (1) gives

$$1 = 3v_a - \frac{10}{8}v_a \longrightarrow v_a = \frac{8}{14}$$

Thus,

$$i_x = \frac{0.5 - v_a}{5} = -1/70\text{ mA} = \underline{-14.28\ \mu\text{A}}$$

$$i_y = \frac{v_o - v_b}{2} + \frac{v_o - v_a}{10} = 0.6(v_o - v_a) = 0.6(\frac{10}{8}v_a - v_a) = \frac{0.6}{4}x\frac{8}{14}\text{ mA} = \underline{85.71\ \mu\text{A}}$$

Chapter 5, Problem 17

Calculate the gain v_o/v_i when the switch in Fig. 5.56 is in:
(a) position 1 (b) position 2 (c) position 3

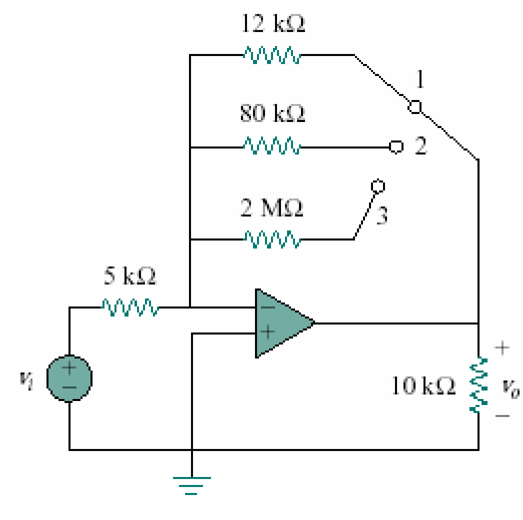


Figure 5.56

Chapter 5, Solution 17.

(a) $$G = \frac{v_o}{v_i} = -\frac{R_2}{R_1} = -\frac{12}{5} = \underline{\mathbf{-2.4}}$$

(b) $$\frac{v_o}{v_i} = -\frac{80}{5} = \underline{\mathbf{-16}}$$

(c) $$\frac{v_o}{v_i} = -\frac{2000}{5} = \underline{\mathbf{-400}}$$

*** Chapter 5, Problem 18.**

For the circuit in Fig. 5.57, find the Thevenin equivalent to the left of terminals a-b. Then calculate the power absorbed by the 20-kΩ resistor. Assume that the op amp is ideal.

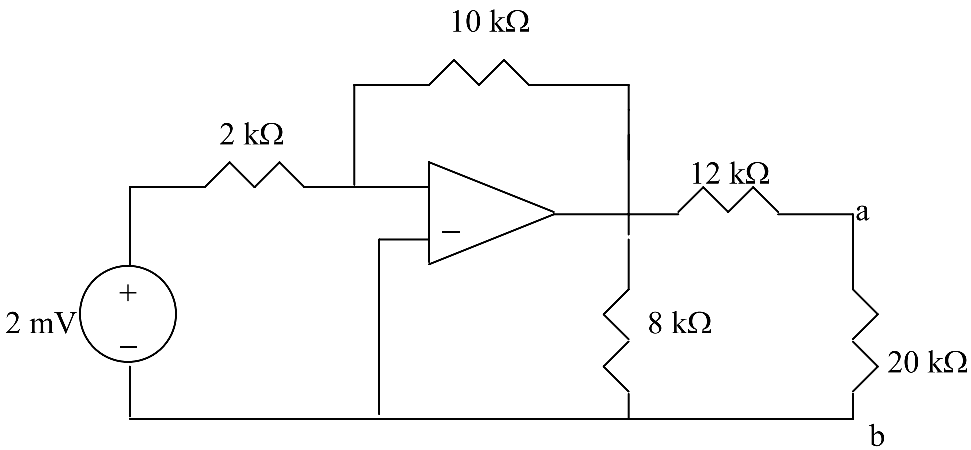
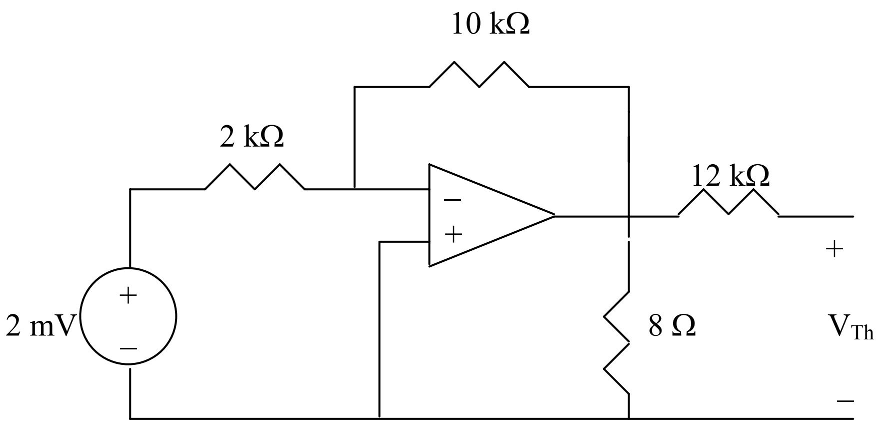


Figure 5.57 For Prob. 5.18.

Chapter 5, Solution 18.

We temporarily remove the 20-kΩ resistor. To find V_{Th}, we consider the circuit below.

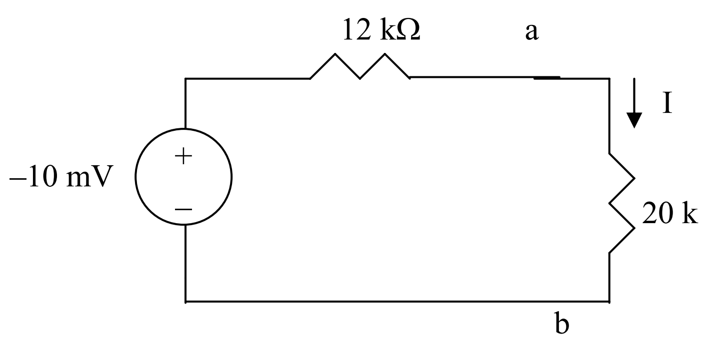


This is an inverting amplifier.

$$V_{Th} = -\frac{10k}{2k}(2mV) = \underline{-10mV}$$

To find R_{Th}, we note that the 8-kΩ resistor is across the output of the op amp which is acting like a voltage source so the only resistance seen looking in is the 12-kΩ resistor.

The Thevenin equivalent with the 20-kΩ resistor is shown below.



$I = -10m/(12k + 20k) = 0.3125x10^{-6}$ A

$p = I^2R = (0.3125x10^{-6})^2x20x10^3 =$ $\underline{\mathbf{1.9531\ nW}}$

Chapter 5, Problem 19

Determine i_o in the circuit of Fig. 5.58.

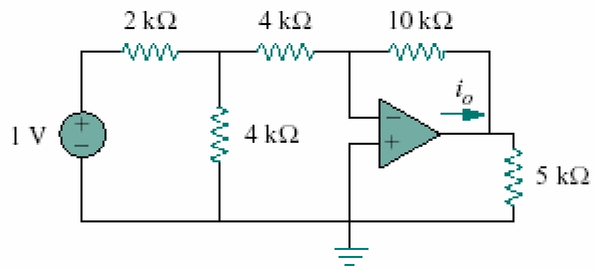
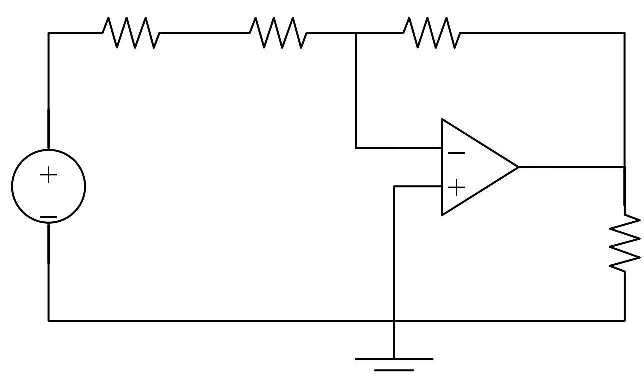


Figure 5.58

Chapter 5, Solution 19.

We convert the current source and back to a voltage source.

$$2\|4 = \frac{4}{3}$$



$$v_o = -\frac{10k}{\left(4+\frac{4}{3}\right)k}\left(\frac{2}{3}\right) = \text{-}1.25\text{V}$$

$$i_o = \frac{v_o}{5k} + \frac{v_o - 0}{10k} = \underline{\mathbf{-0.375mA}}$$

Chapter 5, Problem 20

In the circuit in Fig. 5.59, calculate v_o if $v_s = 0$.

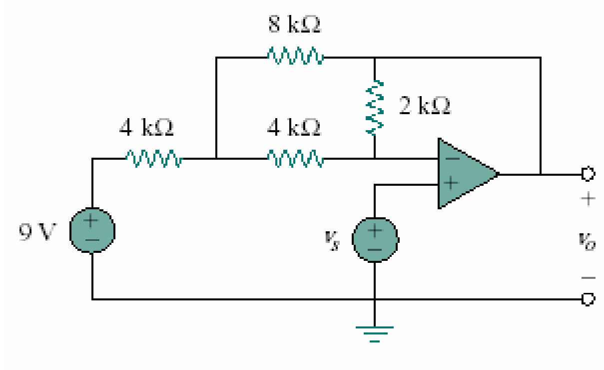
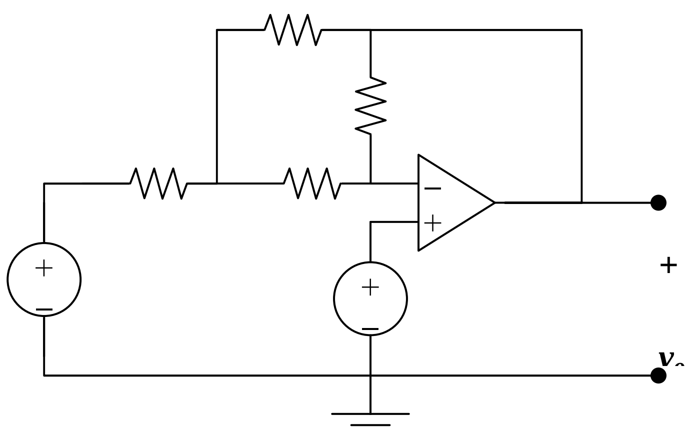


Figure 5.59

Chapter 5, Solution 20.



At node a,

$$\frac{9 - v_a}{4} = \frac{v_a - v_o}{8} + \frac{v_a - v_b}{4} \longrightarrow 18 = 5v_a - v_o - 2v_b \qquad (1)$$

At node b,

$$\frac{v_a - v_b}{4} = \frac{v_b - v_o}{2} \longrightarrow v_a = 3v_b - 2v_o \qquad (2)$$

But $v_b = v_s = 0$; (2) becomes $v_a = -2v_o$ and (1) becomes

$$-18 = -10v_o - v_o \longrightarrow v_o = -18/(11) = \underline{\mathbf{-1.6364V}}$$

Chapter 5, Problem 21.

Calculate v_o in the op amp circuit of Fig. 5.60.

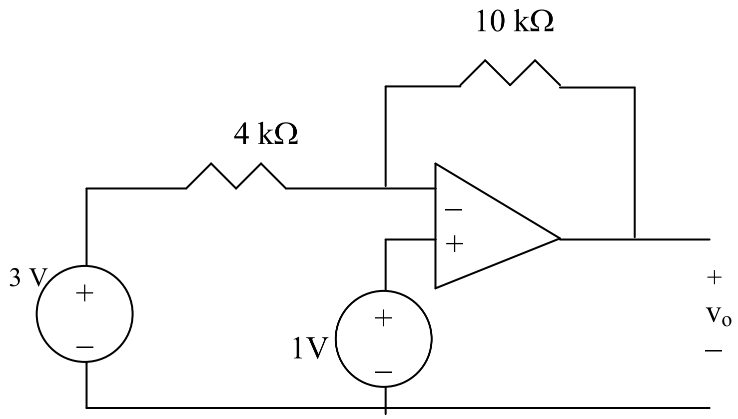


Figure 5.60 For Prob. 5.21.

Chapter 5, Solution 21.

Let the voltage at the input of the op amp be v_a.

$$v_a = 1\text{V}, \quad \frac{3\text{-}v_a}{4k} = \frac{v_a - v_o}{10k} \quad \longrightarrow \quad \frac{3\text{-}1}{4} = \frac{1 - v_o}{10}$$

$v_o = \underline{\mathbf{-4\ V}}$.

Chapter 5, Problem 22

Design an inverting amplifier with a gain of -15.

Chapter 5, Solution 22.

$A_v = -R_f/R_i = -15$.
If $R_i = 10\text{k}\Omega$, then $R_f = \underline{\mathbf{150\ k\Omega.}}$

Chapter 5, Problem 23

For the op amp circuit in Fig. 5.61, find the voltage gain v_o/v_s.

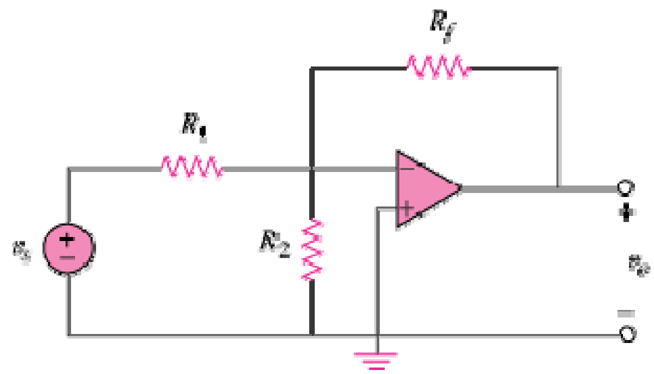


Figure 5.61

Chapter 5, Solution 23

At the inverting terminal, v=0 so that KCL gives

$$\frac{v_s - 0}{R_1} = \frac{0}{R_2} + \frac{0 - v_o}{R_f} \longrightarrow \underline{\frac{v_o}{v_s} = -\frac{R_f}{R_1}}$$

Chapter 5, Problem 24

In the circuit shown in Fig. 5.62, find k in the voltage transfer function $v_o = kv_s$.

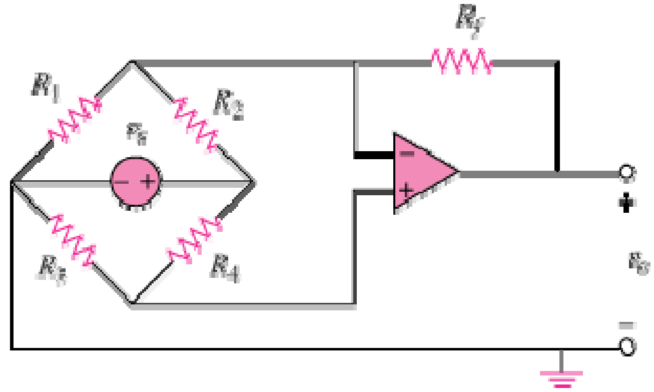
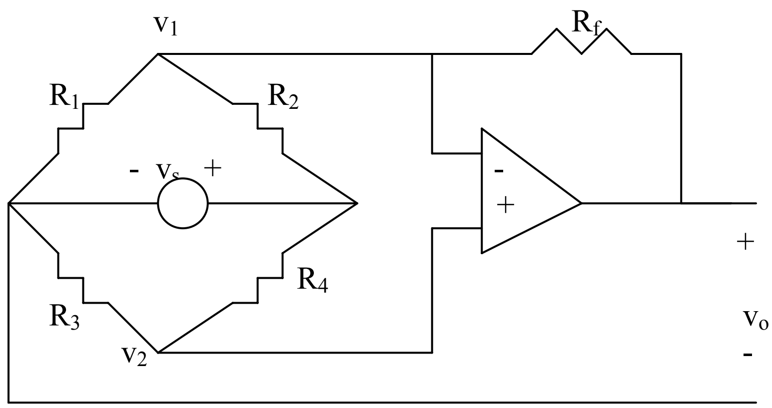


Figure 5.62

Chapter 5, Solution 24



We notice that $v_1 = v_2$. Applying KCL at node 1 gives

$$\frac{v_1}{R_1} + \frac{(v_1 - v_s)}{R_2} + \frac{v_1 - v_o}{R_f} = 0 \longrightarrow \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_f}\right)v_1 - \frac{v_s}{R_2} = \frac{v_o}{R_f} \qquad (1)$$

Applying KCL at node 2 gives

$$\frac{v_1}{R_3} + \frac{v_1 - v_s}{R_4} = 0 \longrightarrow v_1 = \frac{R_3}{R_3 + R_4}v_s \qquad (2)$$

Substituting (2) into (1) yields

$$v_o = R_f\left[\left(\frac{R_3}{R_1} + \frac{R_3}{R_f} - \frac{R_4}{R_2}\right)\left(\frac{R_3}{R_3 + R_4}\right) - \frac{1}{R_2}\right]v_s$$

i.e.

$$k = R_f\left[\left(\frac{R_3}{R_1} + \frac{R_3}{R_f} - \frac{R_4}{R_2}\right)\left(\frac{R_3}{R_3 + R_4}\right) - \frac{1}{R_2}\right]$$

Chapter 5, Problem 25.

Calculate v_o in the op amp circuit of Fig. 5.63.

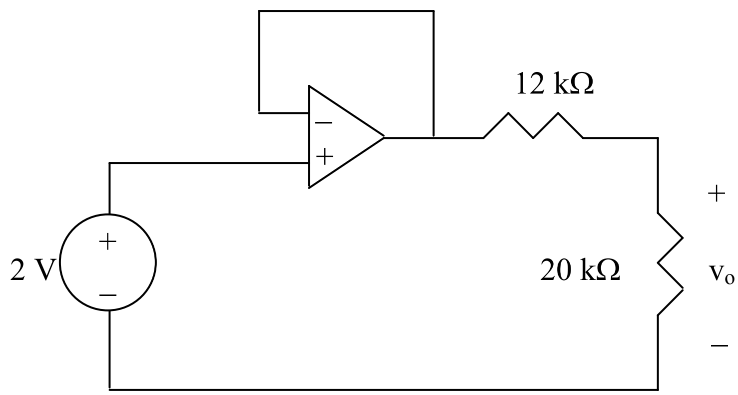


Figure 5.63 For Prob. 5.25.

Chapter 5, Solution 25.

This is a voltage follower. If v_1 is the output of the op amp,

$$v_1 = 2V$$

$$v_o = \frac{20k}{20k+12k} v_1 = \frac{20}{32}(12) = \underline{1.25\ V}$$

Chapter 5, Problem 26

Determine i_o in the circuit of Fig. 5.64.

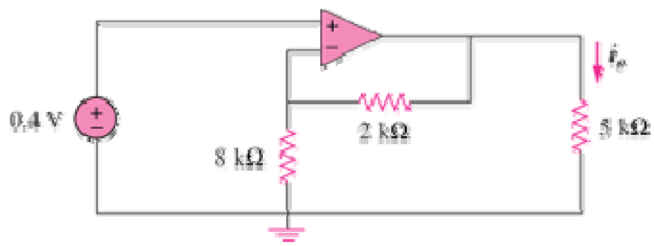
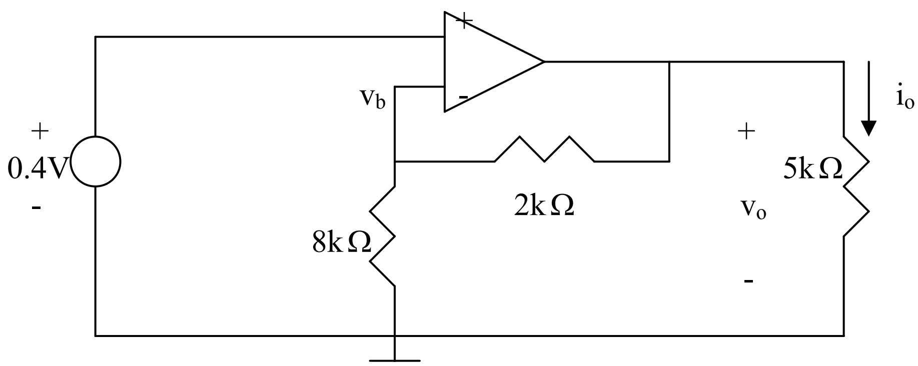


Figure 5.64

Chapter 5, Solution 26



$$v_b = 0.4 = \frac{8}{8+2} v_o = 0.8 v_o \quad \longrightarrow \quad v_o = 0.4/0.8 = 0.5\,\text{V}$$

Hence,

$$i_o = \frac{v_o}{5k} = \frac{0.5}{5k} = \underline{0.1\,\text{mA}}$$

Chapter 5, Problem 27.

Find v_o in the op amp circuit in Fig. 5.65.

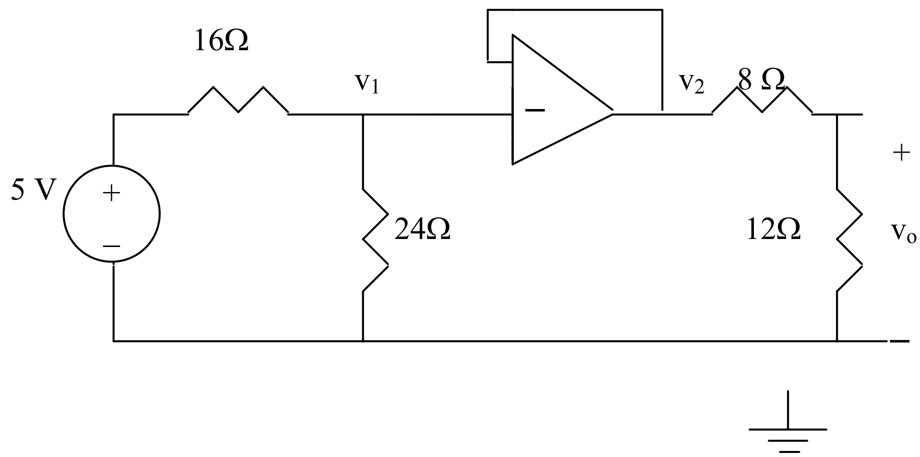


Figure 5.65 For Prob. 5.27.

Chapter 5, Solution 27.

This is a voltage follower.

$$v_1 = \frac{24}{24+16}(5) = 3V, \qquad v_2 = v_1 = 3V$$

$$v_o = \frac{12}{12+8}(3V) = \underline{1.8\ \text{V}}$$

Chapter 5, Problem 28

Find i_o in the op amp circuit of Fig. 5.66.

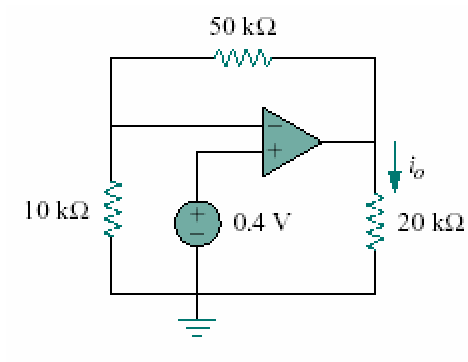
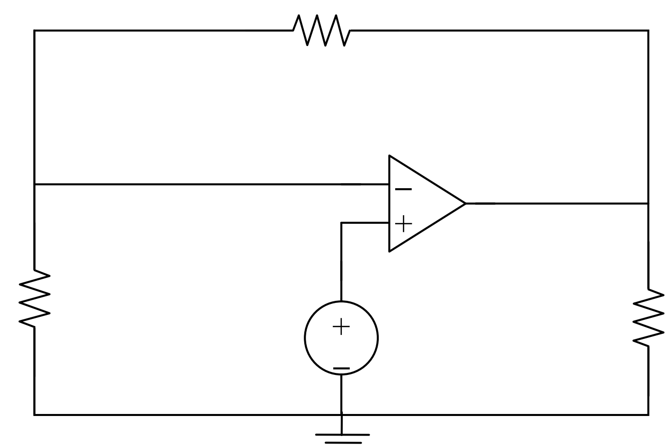


Figure 5.66

Chapter 5, Solution 28.



At node 1, $\frac{0 - v_1}{10k} = \frac{v_1 - v_o}{50k}$

But $v_1 = 0.4V$,

$-5v_1 = v_1 - v_o$, leads to $v_o = 6v_1 = \underline{\mathbf{2.4V}}$

Alternatively, viewed as a noninverting amplifier,

$v_o = (1 + (50/10))\,(0.4V) = \underline{\mathbf{2.4V}}$

$i_o = v_o/(20k) = 2.4/(20k) = \underline{\mathbf{120\ \mu A}}$

Chapter 5, Problem 29

Determine the voltage gain v_o/v_i of the op amp circuit in Fig. 5.67.

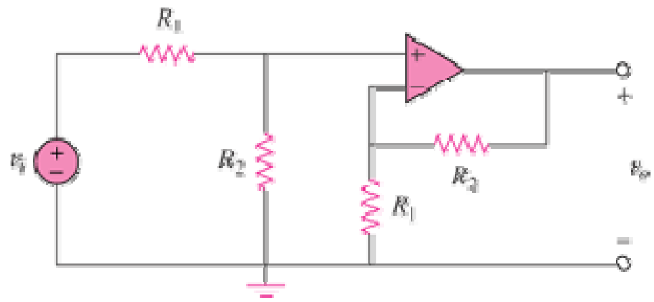
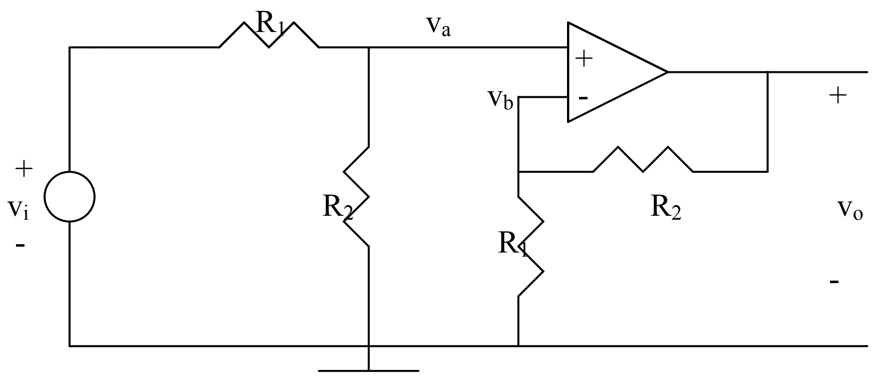


Figure 5.67

Chapter 5, Solution 29



$$v_a = \frac{R_2}{R_1 + R_2} v_i, \qquad v_b = \frac{R_1}{R_1 + R_2} v_o$$

But $v_a = v_b$ $\longrightarrow$ $\dfrac{R_2}{R_1 + R_2} v_i = \dfrac{R_1}{R_1 + R_2} v_o$

Or

$$\underline{\frac{v_o}{v_i} = \frac{R_2}{R_1}}$$

Chapter 5, Problem 30

In the circuit shown in Fig. 5.68, find $\boldsymbol{i_x}$ and the power absorbed by the 20-Ω resistor.

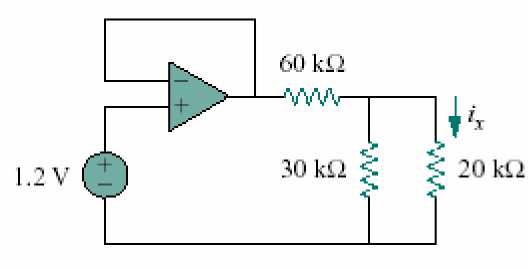


Figure 5.68

Chapter 5, Solution 30.

The output of the voltage becomes

$v_o = v_i = 12$

$30 \| 20 = 12\text{k}\Omega$

By voltage division,

$$v_x = \frac{12}{12+60}(1.2) = 0.2\text{V}$$

$$i_x = \frac{v_x}{20\text{k}} = \frac{0.2}{20\text{k}} = \underline{\mathbf{10\mu A}}$$

$$p = \frac{v_x^2}{R} = \frac{0.04}{20\text{k}} = \underline{\mathbf{2\mu W}}$$

Chapter 5, Problem 31

For the circuit in Fig. 5.69, find $\boldsymbol{i_x}$.

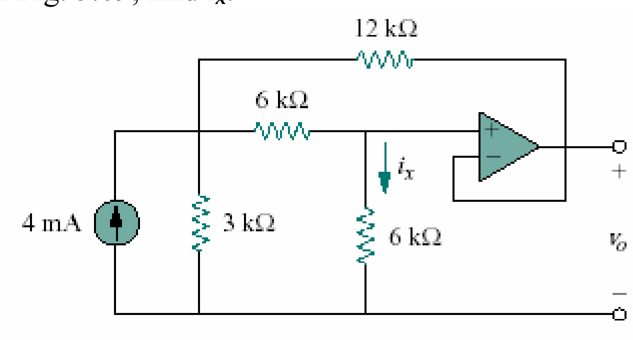
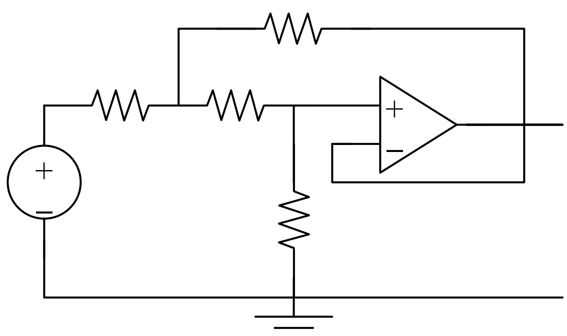


Figure 5.69

Chapter 5, Solution 31.

After converting the current source to a voltage source, the circuit is as shown below:



At node 1,

$$\frac{12 - v_1}{3} = \frac{v_1 - v_o}{6} + \frac{v_1 - v_o}{12} \longrightarrow 48 = 7v_1 - 3v_o \qquad (1)$$

At node 2,

$$\frac{v_1 - v_o}{6} = \frac{v_o - 0}{6} = i_x \longrightarrow v_1 = 2v_o \qquad (2)$$

From (1) and (2),

$$v_o = \frac{48}{11}$$

$$i_x = \frac{v_o}{6k} = \underline{\mathbf{727.2\mu A}}$$

Chapter 5, Problem 32

Calculate i_x and v_o in the circuit of Fig. 5.70. Find the power dissipated by the 60-kΩ resistor.

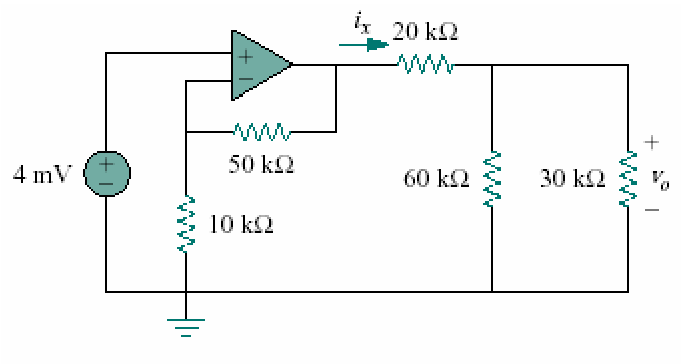


Figure 5.70

Chapter 5, Solution 32.

Let v_x = the voltage at the output of the op amp. The given circuit is a non-inverting amplifier.

$$v_x = \left(1 + \frac{50}{10}\right)(4 \text{ mV}) = 24 \text{ mV}$$

$$60 \| 30 = 20\text{k}\Omega$$

By voltage division,

$$v_o = \frac{20}{20+20} v_x = \frac{v_x}{2} = 12\text{mV}$$

$$i_x = \frac{v_x}{(20+20)\text{k}} = \frac{24\text{mV}}{40\text{k}} = \underline{\mathbf{600nA}}$$

$$p = \frac{v_o^2}{R} = \frac{144\text{x}10^{-6}}{60\text{x}10^3} = \underline{\mathbf{204nW}}$$

Chapter 5, Problem 33

Refer to the op amp circuit in Fig. 5.71. Calculate $\boldsymbol{i_x}$ and the power dissipated by the 3-kΩ resistor.

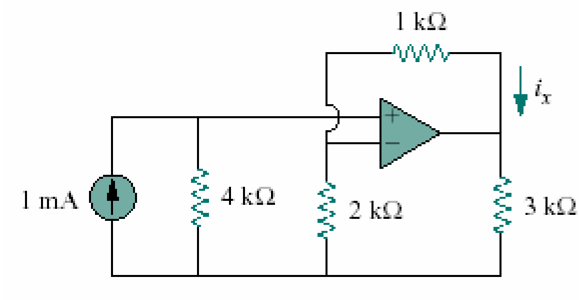
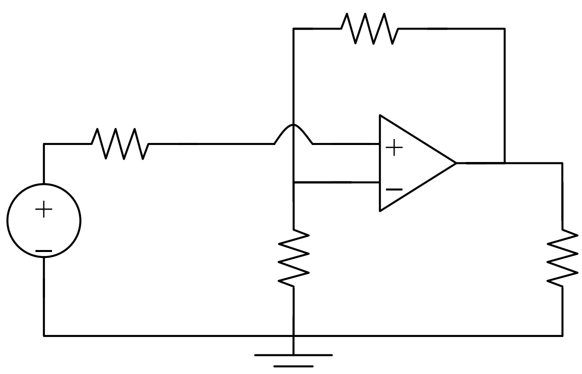


Figure 5.71

Chapter 5, Solution 33.

After transforming the current source, the current is as shown below:



This is a noninverting amplifier.

$$v_o = \left(1 + \frac{1}{2}\right)v_i = \frac{3}{2}v_i$$

Since the current entering the op amp is 0, the source resistor has a OV potential drop. Hence $v_i = 4V$.

$$v_o = \frac{3}{2}(4) = 6V$$

Power dissipated by the 3kΩ resistor is

$$\frac{v_o^2}{R} = \frac{36}{3k} = \underline{\mathbf{12mW}}$$

$$i_x = \frac{v_a - v_o}{R} = \frac{4-6}{1k} = \underline{\mathbf{-2mA}}$$

Chapter 5, Problem 34.

Given the op amp circuit shown in Fig. 5.72, express v_o in terms of v_1 and v_2.

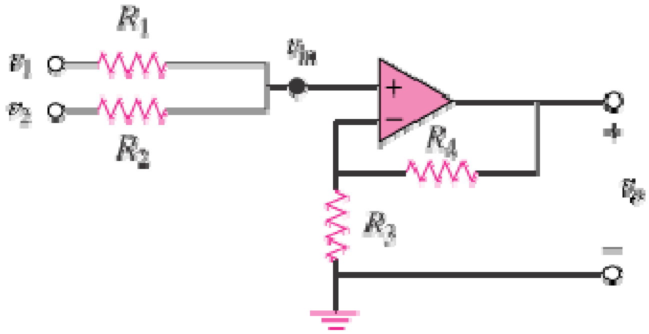


Figure 5.72

Chapter 5, Solution 34

$$\frac{v_1 - v_{in}}{R_1} + \frac{v_1 - v_{in}}{R_2} = 0 \qquad (1)$$

but

$$v_a = \frac{R_3}{R_3 + R_4} v_o \qquad (2)$$

Combining (1) and (2),

$$v_1 - v_a + \frac{R_1}{R_2} v_2 - \frac{R_1}{R_2} v_a = 0$$

$$v_a \left(1 + \frac{R_1}{R_2}\right) = v_1 + \frac{R_1}{R_2} v_2$$

$$\frac{R_3 v_o}{R_3 + R_4}\left(1 + \frac{R_1}{R_2}\right) = v_1 + \frac{R_1}{R_2} v_2$$

$$v_o = \frac{R_3 + R_4}{R_3\left(1 + \frac{R_1}{R_2}\right)}\left(v_1 + \frac{R_1}{R_2} v_2\right)$$

$$\underline{v_O = \frac{R_3 + R_4}{R_3(R_1 + R_2)}(v_1 R_2 + v_2)}$$

Chapter 5, Problem 35

Design a non-inverting amplifier with a gain of 10.

Chapter 5, Solution 35.

$$A_v = \frac{v_o}{v_i} = 1 + \frac{R_f}{R_i} = 10 \longrightarrow R_f = 9R_i$$

If $R_i =$ **<u>10kΩ</u>**, $R_f =$ **<u>90kΩ</u>**

Chapter 5, Problem 36

For the circuit shown in Fig. 5.73, find the Thèvenin equivalent at terminals *a*-*b*. (*Hint*: To find R_{Th}, apply a current source i_o and calculate v_o.)

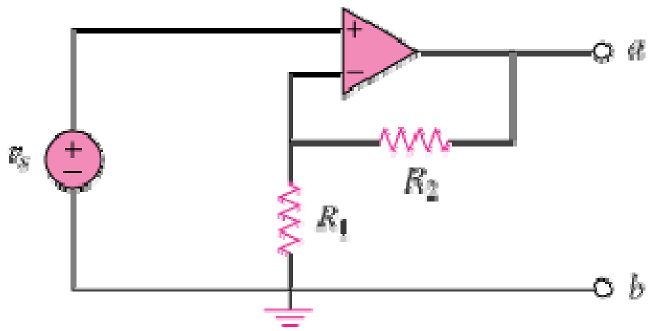


Figure 5.73

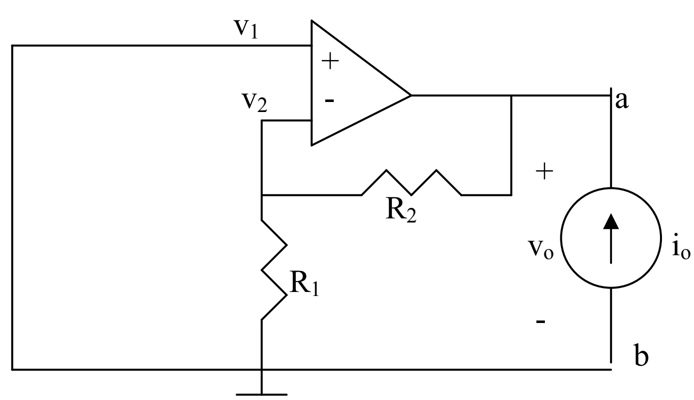
Chapter 5, Solution 36

$$V_{Th} = V_{ab}$$

But $\quad v_s = \dfrac{R_1}{R_1 + R_2} V_{ab}$. Thus,

$$V_{Th} = V_{ab} = \frac{R_1 + R_2}{R_1} v_s = \underline{(1 + \frac{R_2}{R_1}) v_s}$$

To get R_{Th}, apply a current source I_o at terminals a-b as shown below.



Since the noninverting terminal is connected to ground, $v_1 = v_2 = 0$, i.e. no current passes through R_1 and consequently R_2. Thus, v_o=0 and

$$\underline{R_{Th} = \frac{v_o}{i_o} = 0}$$

Chapter 5, Problem 37

Determine the output of the summing amplifier in Fig. 5.74.

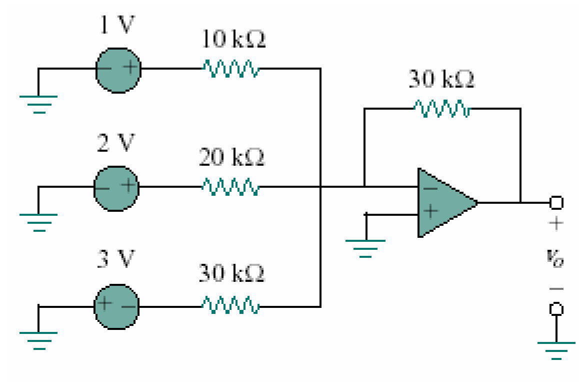


Figure 5.74

Chapter 5, Solution 37.

$$v_o = -\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right]$$

$$= -\left[\frac{30}{10}(1) + \frac{30}{20}(2) + \frac{30}{30}(-3)\right]$$

v_o = **<u>–3V</u>**

Chapter 5, Problem 38

Calculate the output voltage due to the summing amplifier shown in Fig. 5.75.

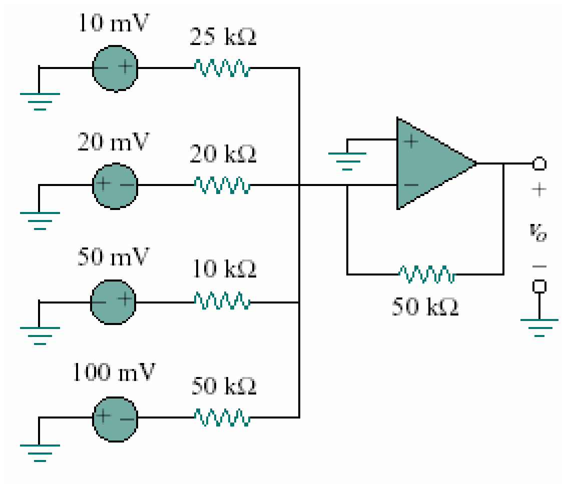


Figure 5.75

Chapter 5, Solution 38.

$$v_o = -\left[\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3 + \frac{R_f}{R_4}v_4\right]$$

$$= -\left[\frac{50}{25}(10) + \frac{50}{20}(-20) + \frac{50}{10}(50) + \frac{50}{50}(-100)\right]$$

= **-120mV**

Chapter 5, Problem 39

For the op amp circuit in Fig. 5.76, determine the value of v_2 in order to make v_o = -16.5 V.

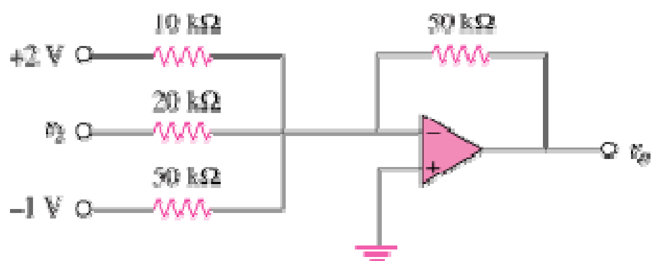


Figure 5.76

Chapter 5, Solution 39

This is a summing amplifier.

$$v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right) = -\left(\frac{50}{10}(2) + \frac{50}{20}v_2 + \frac{50}{50}(-1)\right) = -9 - 2.5v_2$$

Thus,

$$v_o = -16.5 = -9 - 2.5v_2 \quad \longrightarrow \quad \underline{v_2 = 3\text{ V}}$$

Chapter 5, Problem 40.

Find v_o in terms of v_1, v_2, and v_3, in the circuit of Fig. 5.77.

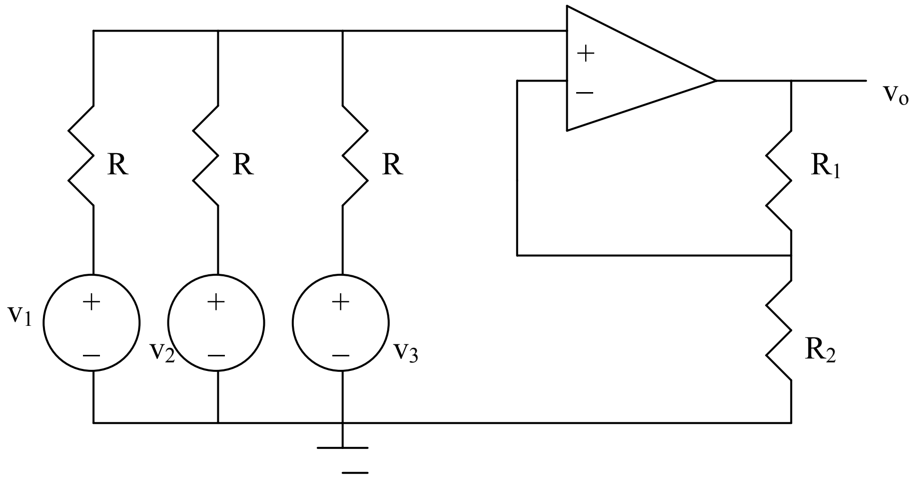


Figure 5.77 For Prob. 5.40.

Chapter 5, Solution 40.

Applying KCL at node a, where node a is the input to the op amp.

$$\frac{v_1 - v_a}{R} + \frac{v_2 - v_a}{R} + \frac{v_3 - v_a}{R} = 0 \quad \text{or} \quad v_a = (v_1 + v_2 + v_3)/3$$

$$v_o = (1 + R_1/R_2)v_a = \underline{\mathbf{(1 + R_1/R_2)(v_1 + v_2 + v_3)/3}}.$$

Chapter 5, Problem 41

An *averaging amplifier* is a summer that provides an output equal to the average of the inputs. By using proper input and feedback resistor values, one can get

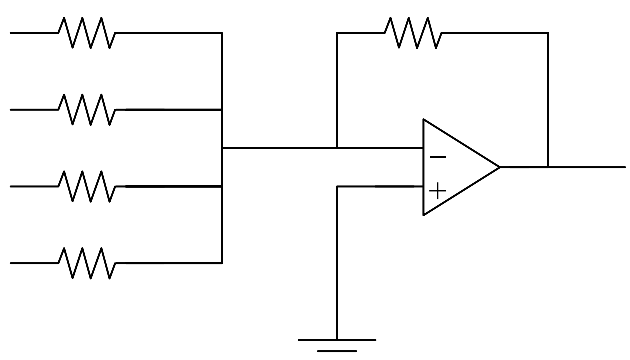
$$-v_{out} = \frac{1}{4}(v_1 + v_2 + v_3 + v_4)$$

Using a feedback resistor of 10 kΩ, design an averaging amplifier with four inputs.

Chapter 5, Solution 41.

$$R_f/R_i = 1/(4) \longrightarrow R_i = 4R_f = 40\text{k}\Omega$$

The averaging amplifier is as shown below:



Chapter 5, Problem 42

A three-input summing amplifier has input resistors with $R_1 = R_2 = R_3 = 30$ kΩ. To produce an averaging amplifier, what value of feedback resistor is needed?

Chapter 5, Solution 42

$$R_f = \frac{1}{3}R_1 = \underline{10\text{ k}\Omega}$$

Chapter 5, Problem 43

A four-input summing amplifier has $R_1 = R_2 = R_3 = R_4 = 12$ kΩ. What value of feedback resistor is needed to make it an averaging amplifier?

Chapter 5, Solution 43.

In order for

$$v_o = \left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \frac{R_f}{R_3} v_3 + \frac{R_f}{R_4} v_4 \right)$$

to become

$$v_o = -\frac{1}{4}(v_1 + v_2 + v_3 + v_4)$$

$$\frac{R_f}{R_i} = \frac{1}{4} \longrightarrow R_f = \frac{R_i}{4} = \frac{12}{4} = \underline{\mathbf{3k\Omega}}$$

Chapter 5, Problem 44

Show that the output voltage v_o of the circuit in Fig. 5.78 is

$$v_o = \frac{(R_3 + R_4)}{R_3(R_1 + R_2)}(R_2 v_1 + R_1 v_2)$$

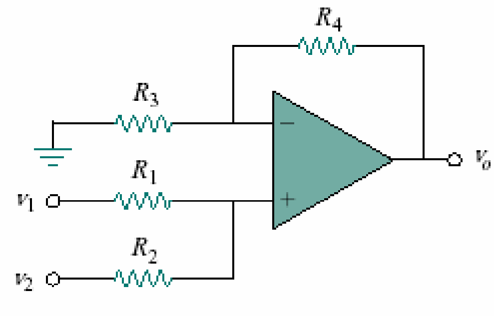
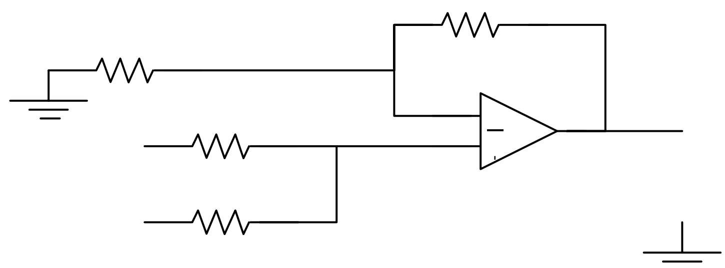


Figure 5.78

Chapter 5, Solution 44.



At node b, $\frac{v_b - v_1}{R_1} + \frac{v_b - v_2}{R_2} = 0 \longrightarrow v_b = \frac{\frac{v_1}{R_1} + \frac{v_2}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$ (1)

At node a, $\frac{0 - v_a}{R_3} = \frac{v_a - v_o}{R_4} \longrightarrow v_a = \frac{v_o}{1 + R_4 / R_3}$ (2)

But $v_a = v_b$. We set (1) and (2) equal.

$$\frac{v_o}{1 + R_4 / R_3} = \frac{R_2 v_1 + R_1 v_2}{R_1 + R_2}$$

or

$$v_o = \underline{\frac{(R_3 + R_4)}{R_3(R_1 + R_2)}(R_2 v_1 + R_1 v_2)}$$

Chapter 5, Problem 45

Design an op amp circuit to perform the following operation:

$$v_o = 3v_1 - 2v_2$$

All resistances must be $\leq$ 100 kΩ.

Chapter 5, Solution 45.

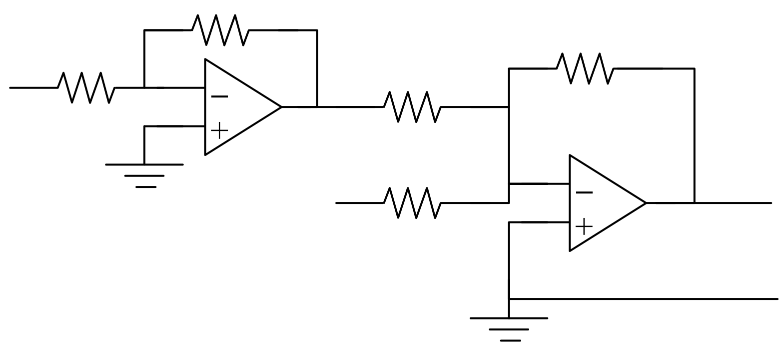
This can be achieved as follows:

$$v_o = -\left[\frac{R}{R/3}(-v_1) + \frac{R}{R/2}v_2\right]$$

$$= -\left[\frac{R_f}{R_1}(-v_1) + \frac{R_f}{R_2}v_2\right]$$

i.e. $R_f = R$, $R_1 = R/3$, and $R_2 = R/2$

Thus we need an inverter to invert v_1, and a summer, as shown below ($R<100k\Omega$).



Chapter 5, Problem 46

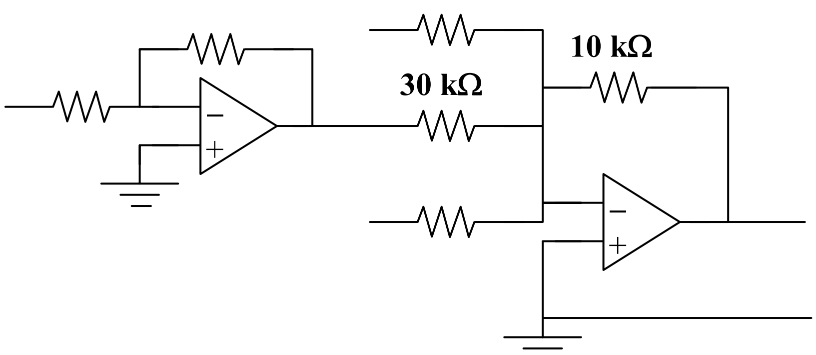
Using only two op amps, design a circuit to solve

$$-v_{out} = \frac{v_1 - v_2}{3} + \frac{v_3}{2}$$

Chapter 5, Solution 46.

$$-v_o = \frac{v_1}{3} + \frac{1}{3}(-v_2) + \frac{1}{2}v_3 = \frac{R_f}{R_1}v_1 + \frac{R_x}{R_2}(-v_2) + \frac{R_f}{R_3}v_3$$

i.e. $R_3 = 2R_f$, $R_1 = R_2 = 3R_f$. To get $-v_2$, we need an inverter with $R_f = R_i$. If $R_f = 10k\Omega$, a solution is given below.



Chapter 5, Problem 47.

The circuit in Fig. 5.79 is for a difference amplifier. Find v_o given that v_1 =1V and v_2 = 2V.

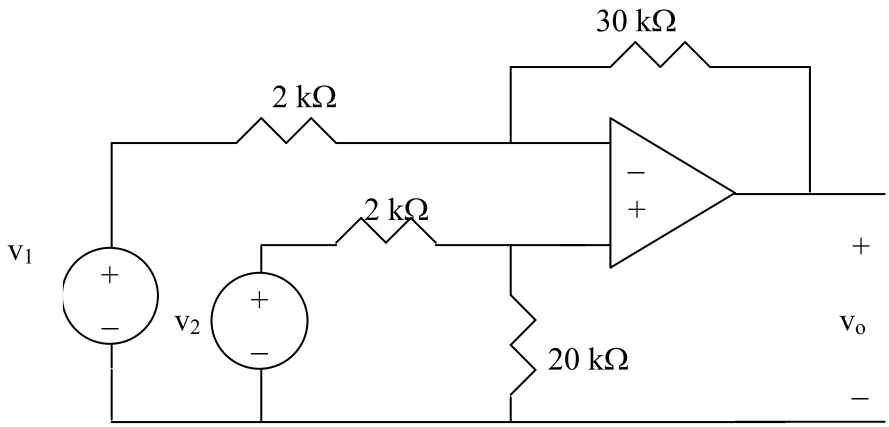


Figure 5.79 For Prob. 5.47.

Chapter 5, Solution 47.

Using eq. (5.18), $R_1 = 2k\Omega,\ R_2 = 30k\Omega,\ R_3 = 2k\Omega,\ R_4 = 20k\Omega$

$$v_o = \frac{30(1+2/30)}{2(1+2/20)}V_2 - \frac{30}{2}V_1 = \frac{32}{2.2}(2) - 15(1) = \underline{14.09\ \text{V}}$$

Chapter 5, Problem 48

The circuit in Fig. 5.80 is a differential amplifier driven by a bridge. Find v_o.

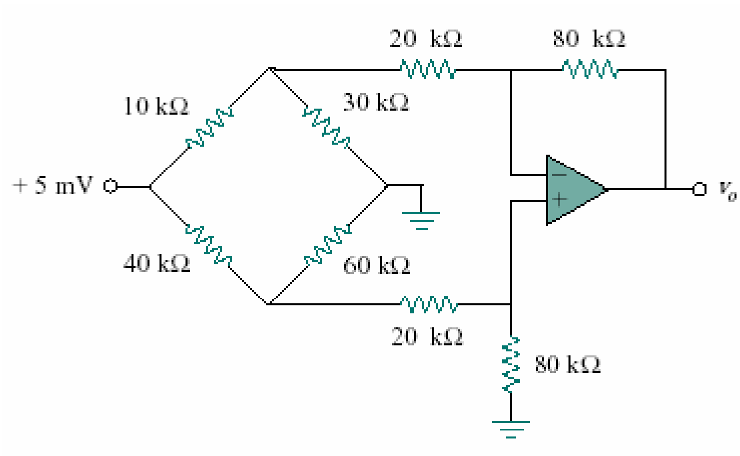
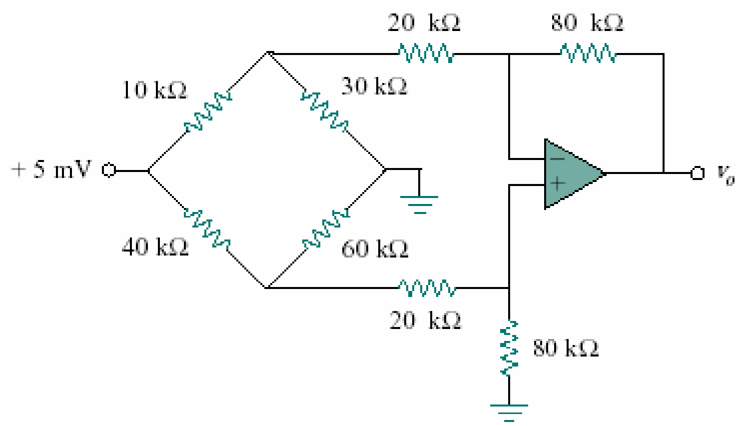


Figure 5.80

Chapter 5, Solution 48.

We can break this problem up into parts. The 5 mV source separates the lower circuit from the upper. In addition, there is no current flowing into the input of the op amp which means we now have the 40-kohm resistor in series with a parallel combination of the 60-kohm resistor and the equivalent 100-kohm resistor.



Thus, $40k + (60x100k)/(160) = 77.5k$

which leads to the current flowing through this part of the circuit,

$$i = 5m/77.5k = 6.452x10^{-8}$$

The voltage across the 60k and equivalent 100k is equal to,

$$v = ix37.5k = 2.419mV$$

We can now calculate the voltage across the 80-kohm resistor.

$$v_{80} = 0.8x2.419m = 1.9352mV$$

which is also the voltage at both inputs of the op amp and the voltage between the 20-kohm and 80-kohm resistors in the upper circuit. Let v_1 be the voltage to the left of the 20-kohm resistor of the upper circuit and we can write a node equation at that node.

$$(v_1–5m)/(10k) + v_1/30k + (v_1–1.9352m)/20k = 0$$

or $$6v_1 – 30m + 2v_1 + 3v_1 – 5.806m = 0$$

or $$v_1 = 35.806m/11 = 3.255mV$$

The current through the 20k-ohm resistor, left to right, is,

$$i_{20} = (3.255m–1.9352m)/20k = 6.599x10^{-8} \text{ A}$$

thus, $$v_o = 1.9352m – 6.599x10^{-8}x80k$$
$$= 1.9352m – 5.2792m = \underline{\mathbf{–3.344\ mV}}.$$

Chapter 5, Problem 49

Design a difference amplifier to have a gain of 2 and a common mode input resistance of 10 kΩ at each input.

Chapter 5, Solution 49.

$$R_1 = R_3 = 10k\Omega,\ R_2/(R_1) = 2$$

i.e. $R_2 = 2R_1 = 20k\Omega = R_4$

Verify: $$v_o = \frac{R_2}{R_1}\frac{1+R_1/R_2}{1+R_3/R_4}v_2 - \frac{R_2}{R_1}v_1$$

$$= 2\frac{(1+0.5)}{1+0.5}v_2 - 2v_1 = 2(v_2 - v_1)$$

Thus, $R_1 = R_3 =$ **10kΩ**, $R_2 = R_4 =$ **20kΩ**

Chapter 5, Problem 50

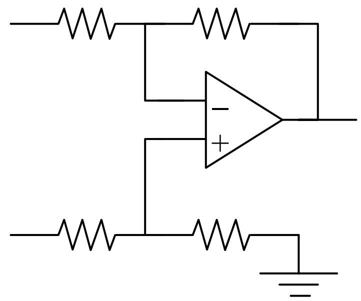
Design a circuit to amplify the difference between two inputs by 2.

(a) Use only one op amp.

(b) Use two op amps.

Chapter 5, Solution 50.

(a) We use a difference amplifier, as shown below:



$$v_o = \frac{R_2}{R_1}(v_2 - v_1) = 2(v_2 - v_1), \text{ i.e. } R_2/R_1 = 2$$

If R_1 = **10 kΩ** then R_2 = **20kΩ**

(b) We may apply the idea in Prob. 5.35.

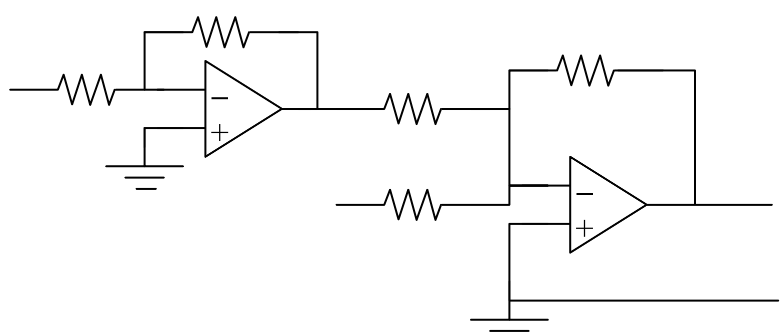
$$v_0 = 2v_1 - 2v_2$$

$$= -\left[\frac{R}{R/2}(-v_1) + \frac{R}{R/2}v_2\right]$$

$$= -\left[\frac{R_f}{R_1}(-v_1) + \frac{R_f}{R_2}v_2\right]$$

i.e. $R_f = R$, $R_1 = R/2 = R_2$

We need an inverter to invert v_1 and a summer, as shown below. We may let R = 10kΩ.

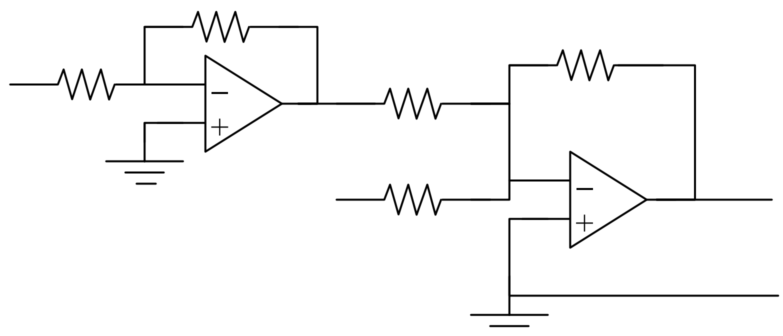


Chapter 5, Problem 51

Using two op amps, design a subtractor.

Chapter 5, Solution 51.

We achieve this by cascading an inverting amplifier and two-input inverting summer as shown below:



Verify:

$v_o = -v_a - v_2$

But $v_a = -v_1$. Hence

$v_o = v_1 - v_2$.

Chapter 5, Problem 52

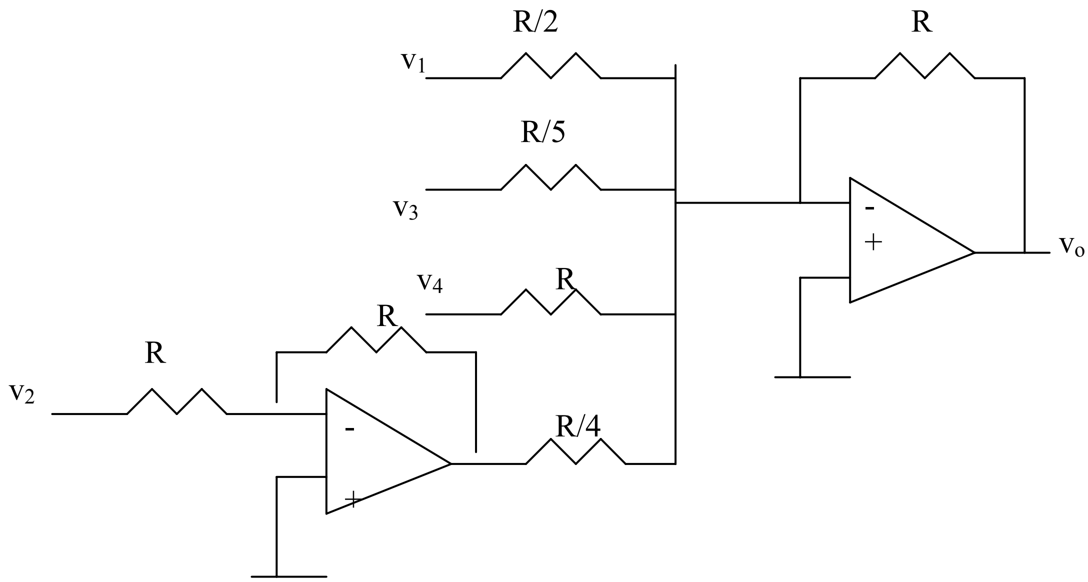
Design an op amp circuit such that

$v_o = -2v_1 + 4v_2 - 5v_3 - v_4$

Let all the resistors be in the range of 5 to 100 kΩ.

Chapter 5, Solution 52

A summing amplifier shown below will achieve the objective. An inverter is inserted to invert v_2. Let $R = 10\ \text{k}\Omega$.



Chapter 5, Problem 53

The ordinary difference amplifier for fixed-gain operation is shown in Fig. 5.81(a). It is simple and reliable unless gain is made variable. One way of providing gain adjustment without losing simplicity and accuracy is to use the circuit in Fig. 5.81(b). Another way is to use the circuit in Fig. 5.81(c). Show that:

(a) for the circuit in Fig. 5.81(a),

$$\frac{v_o}{v_i} = \frac{R_2}{R_1}$$

(b) for the circuit in Fig. 5.81(b),

$$\frac{v_o}{v_i} = \frac{R_2}{R_1} \frac{1}{1 + \frac{R_1}{2R_G}}$$

(c) for the circuit in Fig. 5.81(c),

$$\frac{v_o}{v_i} = \frac{R_2}{R_1}\left(1 + \frac{R_2}{2R_G}\right)$$

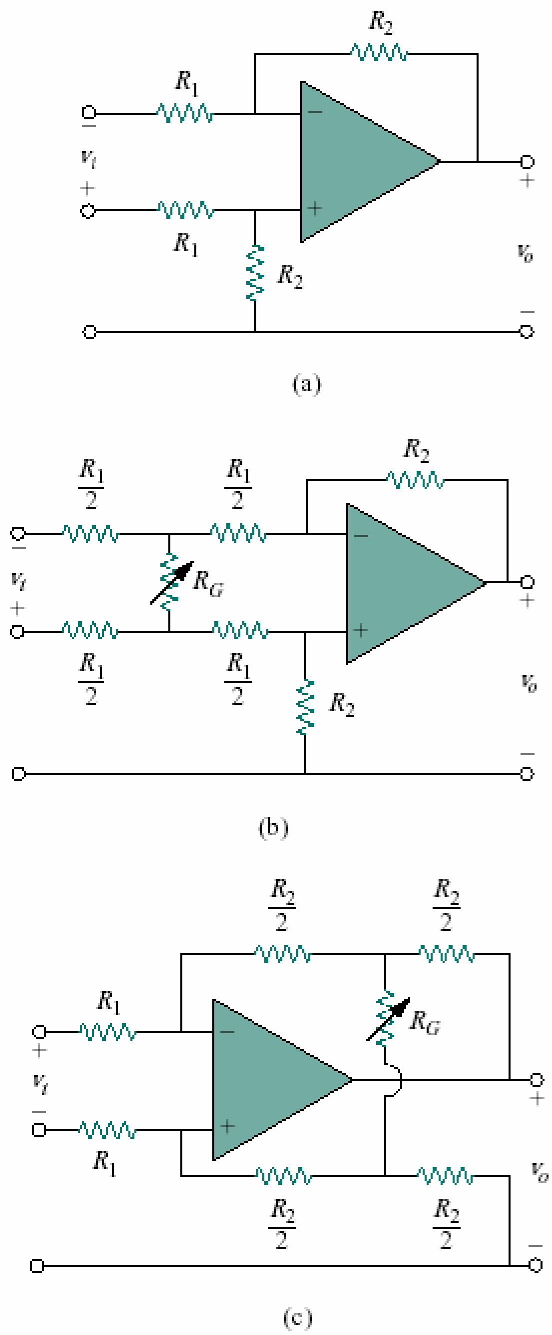
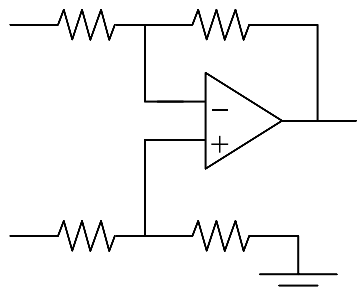

R2
R1
vi
R1
R2
vo
(a)
R1/2
R1/2
R2
RG
R1/2
R1/2
R2
vi
vo
(b)
R2/2
R2/2
R1
RG
vi
R1
R2/2
R2/2
vo
(c)

Figure 5.81

Chapter 5, Solution 53.

(a)



At node a,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2} \longrightarrow v_a = \frac{R_2 v_1 + R_1 v_o}{R_1 + R_2} \qquad (1)$$

At node b, $$v_b = \frac{R_2}{R_1 + R_2} v_2 \qquad (2)$$

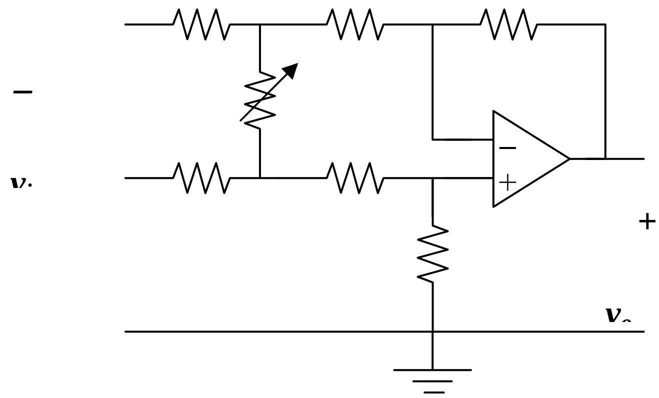
But $v_a = v_b$. Setting (1) and (2) equal gives

$$\frac{R_2}{R_1 + R_2} v_2 = \frac{R_2 v_1 + R_1 v_o}{R_1 + R_2}$$

$$v_2 - v_1 = \frac{R_1}{R_2} v_o = v_i$$

$$\frac{v_o}{v_i} = \underline{\frac{R_2}{R_1}}$$

(b)



At node A, $$\frac{v_1 - v_A}{R_1/2} + \frac{v_B - v_A}{R_g} = \frac{v_A - v_a}{R_1/2}$$

or $$v_1 - v_A + \frac{R_1}{2R_g}\left(v_B - v_A\right) = v_A - v_a \qquad (1)$$

At node B, $$\frac{v_2 - v_B}{R_1/2} = \frac{v_B - v_A}{R_1/2} + \frac{v_B - v_b}{R_g}$$

or $$v_2 - v_B - \frac{R_1}{2R_g}(v_B - v_A) = v_B - v_b \qquad (2)$$

Subtracting (1) from (2),

$$v_2 - v_1 - v_B + v_A - \frac{2R_1}{2R_g}\left(v_B - v_A\right) = v_B - v_A - v_b + v_a$$

Since, $v_a = v_b$,

$$\frac{v_2 - v_1}{2} = \left(1 + \frac{R_1}{2R_g}\right)\left(v_B - v_A\right) = \frac{v_i}{2}$$

or $$v_B - v_A = \frac{v_i}{2} \cdot \frac{1}{1 + \frac{R_1}{2R_g}} \qquad (3)$$

But for the difference amplifier,

$$v_o = \frac{R_2}{R_1/2}\left(v_B - v_A\right)$$

or $$v_B - v_A = \frac{R_1}{2R_2} v_o \qquad (4)$$

Equating (3) and (4), $$\frac{R_1}{2R_2} v_o = \frac{v_i}{2} \cdot \frac{1}{1 + \dfrac{R_1}{2R_g}}$$

$$\underline{\frac{v_o}{v_i} = \frac{R_2}{R_1} \cdot \frac{1}{1 + \dfrac{R_1}{2R_g}}}$$

(c) At node a, $\dfrac{v_1 - v_a}{R_1} = \dfrac{v_a - v_A}{R_2/2}$

$$v_1 - v_a = \frac{2R_1}{R_2}v_a - \frac{2R_1}{R_2}v_A \qquad (1)$$

At node b, $$v_2 - v_b = \frac{2R_1}{R_2}v_b - \frac{2R_1}{R_2}v_B \qquad (2)$$

Since $v_a = v_b$, we subtract (1) from (2),

$$v_2 - v_1 = \frac{-2R_1}{R_2}(v_B - v_A) = \frac{v_i}{2}$$

or $$v_B - v_A = \frac{-R_2}{2R_1}v_i \qquad (3)$$

At node A,

$$\frac{v_a - v_A}{R_2/2} + \frac{v_B - v_A}{R_g} = \frac{v_A - v_o}{R/2}$$

$$v_a - v_A + \frac{R_2}{2R_g}(v_B - v_A) = v_A - v_o \qquad (4)$$

At node B, $\dfrac{v_b - v_B}{R/2} - \dfrac{v_B - v_A}{R_g} = \dfrac{v_B - 0}{R/2}$

$$v_b - v_B - \frac{R_2}{2R_g}(v_B - v_A) = v_B \qquad (5)$$

Subtracting (5) from (4),

$$v_B - v_A + \frac{R_2}{R_g}(v_B - v_A) = v_A - v_B - v_o$$

$$2(v_B - v_A)\left(1 + \frac{R_2}{2R_g}\right) = -v_o \qquad (6)$$

Combining (3) and (6),

$$\frac{-R_2}{R_1}v_i\left(1 + \frac{R_2}{2R_g}\right) = -v_o$$

$$\underline{\frac{v_o}{v_i} = \frac{R_2}{R_1}\left(1 + \frac{R_2}{2R_g}\right)}$$

Chapter 5, Problem 54.

Determine the voltage transfer ratio v_o/v_s in the op amp circuit of Fig. 5.82, where R =10 kΩ.

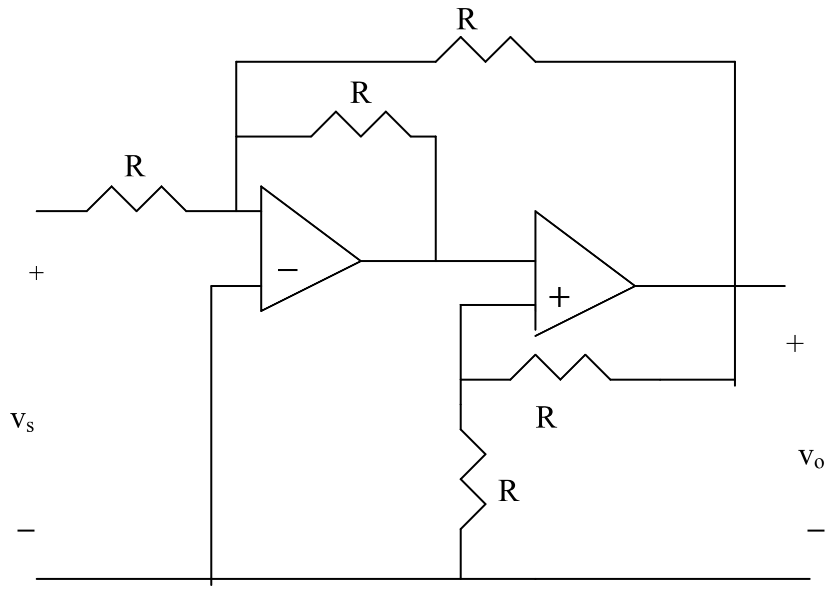


Figure 5.82 For Prob. 5.54.

Chapter 5, Solution 54.
The first stage is a summer (please note that we let the output of the first stage be v_1).

$$v_1 = -\left(\frac{R}{R}v_s + \frac{R}{R}v_o\right) = -v_s - v_o$$

The second stage is a noninverting amplifier

$$v_o = (1 + R/R)v_1 = 2v_1 = 2(-v_s - v_o) \quad \text{or } 3v_o = -2v_s$$

$$v_o/v_s = \underline{\mathbf{-0.6667}}.$$

Chapter 5, Problem 55

In a certain electronic device, a three-stage amplifier is desired, whose overall voltage gain is 42 dB. The individual voltage gains of the first two stages are to be equal, while the gain of the third is to be one-fourth of each of the first two. Calculate the voltage gain of each.

Chapter 5, Solution 55.

Let $A_1 = k$, $A_2 = k$, and $A_3 = k/(4)$

$A = A_1A_2A_3 = k^3/(4)$

$20\text{Log}_{10}A = 42$

$\text{Log}_{10}A = 2.1 \longrightarrow A = 10^{2.1} = 125.89$

$k^3 = 4A = 503.57$

$k = \sqrt[3]{503.57} = 7.956$

Thus $A_1 = A_2 = \underline{\mathbf{7.956}}$, $A_3 = \underline{\mathbf{1.989}}$

Chapter 5, Problem 56.

Calculate the gain of the op amp circuit shown in Fig. 5.83.

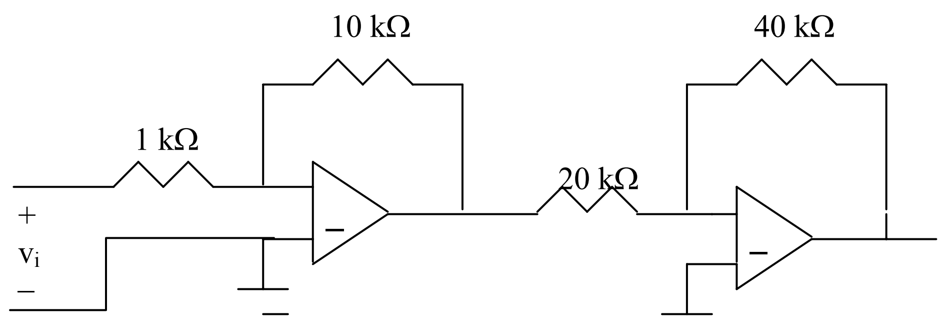


Figure 5.83 **For Prob. 5.56.**

Chapter 5, Solution 56.

Each stage is an inverting amplifier. Hence.

$$\frac{v_o}{v_s} = \left(-\frac{10}{1}\right)\left(-\frac{40}{20}\right) = \underline{20}$$

Chapter 5, Problem 57.

Find v_o in the op amp circuit of Fig. 5.84.

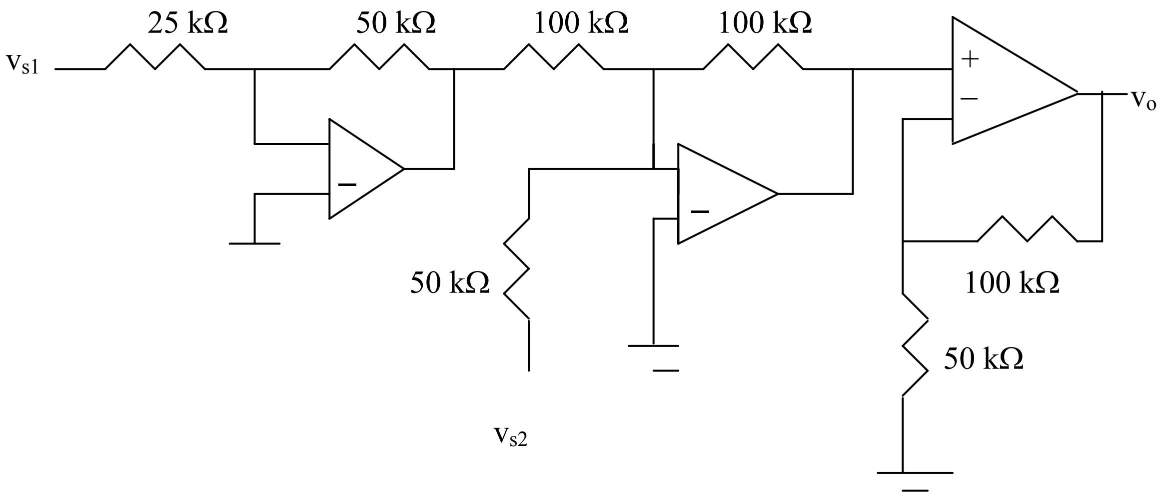


Figure 5.84 For Prob. 5.57.

Chapter 5, Solution 57.

Let v_1 be the output of the first op amp and v_2 be the output of the second op amp.

The first stage is an inverting amplifier.

$$v_1 = -\frac{50}{25}v_{s1} = -2v_{s1}$$

The second state is a summer.

$$v_2 = -(100/50)v_{s2} - (100/100)v_1 = -2v_{s2} + 2v_{s1}$$

The third state is a noninverting amplifier

$$v_o = (1+\frac{100}{50})v_2 = 3v_2 = \underline{6v_{s1} - 6v_{s2}}$$

Chapter 5, Problem 58

Calculate i_o in the op amp circuit of Fig. 5.85.

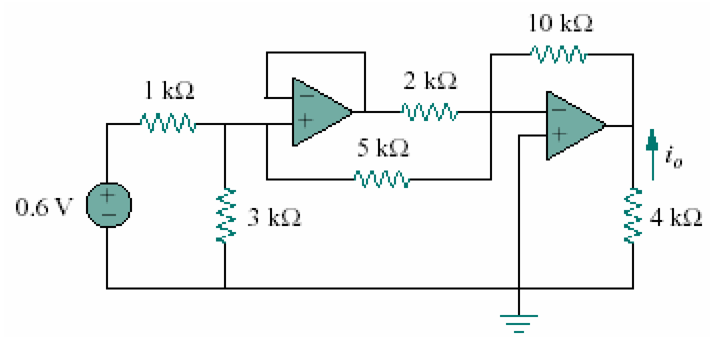


Figure 5.85

Chapter 5, Solution 58.

Looking at the circuit, the voltage at the right side of the 5-kΩ resistor must be at 0V if the op amps are working correctly. Thus the 1-kΩ is in series with the parallel combination of the 3-kΩ and the 5-kΩ. By voltage division, the input to the voltage follower is:

$$v_1 = \frac{3\|5}{1+3\|5}(0.6) = 0.3913\text{V} = \text{to the output of the first op amp.}$$

Thus

$$v_o = -10((0.3913/5)+(0.3913/2)) = -2.739 \text{ V.}$$

$$i_o = \frac{0 - v_o}{4k} = \underline{\mathbf{0.6848 \text{ mA}}}$$

Chapter 5, Problem 59.

In the op amp circuit of Fig. 5.86, determine the voltage gain v_o/v_s. Take R = 10 kΩ.

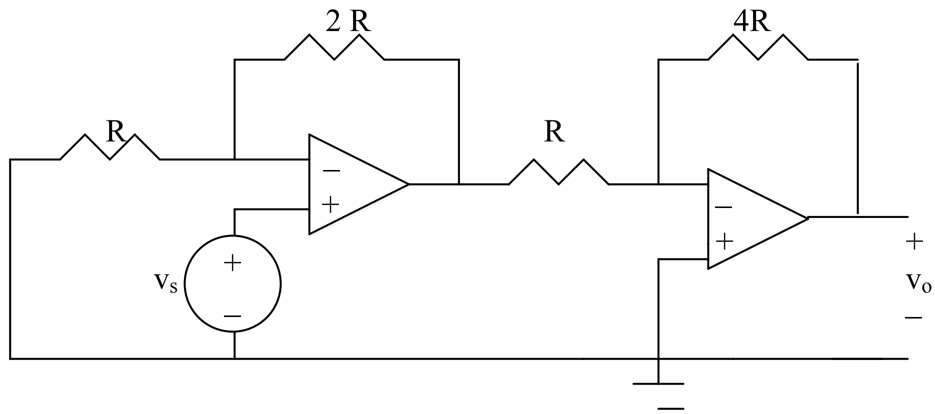


Figure 5.86 For Prob. 5.59.

Chapter 5, Solution 59.

The first stage is a noninverting amplifier. If v_1 is the output of the first op amp,

$$v_1 = (1 + 2R/R)v_s = 3v_s$$

The second stage is an inverting amplifier

$$v_o = -(4R/R)v_1 = -4v_1 = -4(3v_s) = -12v_s$$

$$v_o/v_s = \underline{\mathbf{-12}}.$$

Chapter 5, Problem 60.

Calculate v_o/v_i in the op amp circuit in Fig. 5.87.

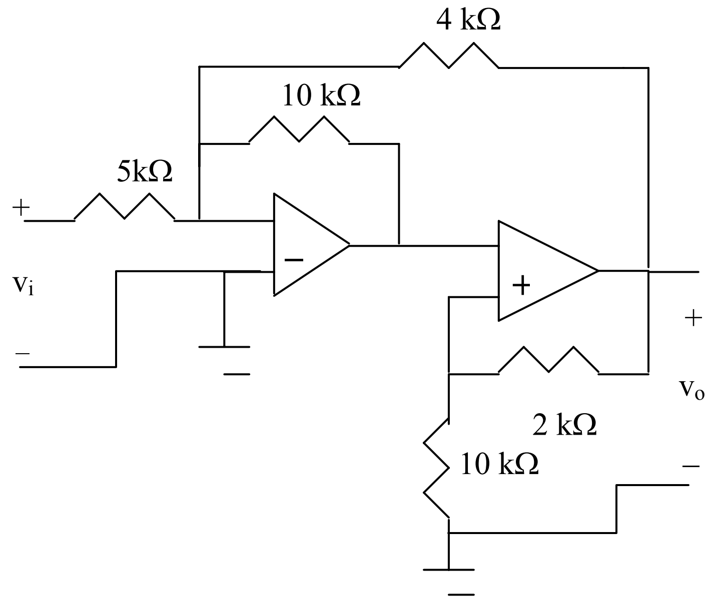


Figure 5.87 For Prob. 5.60.

Chapter 5, Solution 60.

The first stage is a summer. Let V_1 be the output of the first stage.

$$v_1 = -\frac{10}{5}v_i - \frac{10}{4}v_o \quad \longrightarrow \quad v_1 = -2v_i - 2.5v_o \qquad (1)$$

By voltage division,

$$v_1 = \frac{10}{10+2}v_o = \frac{5}{6}v_o \qquad (2)$$

Combining (1) and (2),

$$\frac{5}{6}v_o = -2v_1 - 2.5v_0 \quad \longrightarrow \quad \frac{10}{3}v_0 = -2v_i$$

$$\frac{v_o}{v_i} = -6/10 = \underline{-0.6}$$

Chapter 5, Problem 61.

Determine v_o in the circuit of Fig. 5.88.

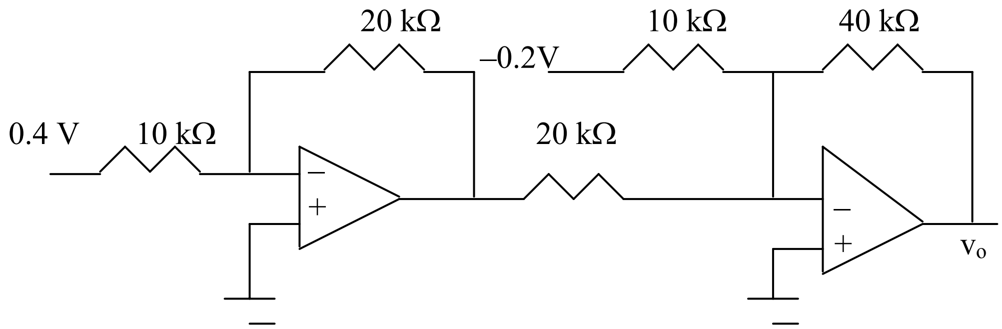


Figure 5.88 **For Prob. 5.61.**

Chapter 5, Solution 61.

The first op amp is an inverter. If v_1 is the output of the first op amp,

$$v_1 = -\frac{200}{100}(0.4) = -0.8V$$

The second op amp is a summer

$$V_o = \frac{-40}{10}(0.2) - \frac{40}{20}(0.8) = 0.8 + 1.6 = \underline{2.4\ \text{V}}$$

Chapter 5, Problem 62

Obtain the closed-loop voltage gain v_o/v_i of the circuit in Fig. 5.89.

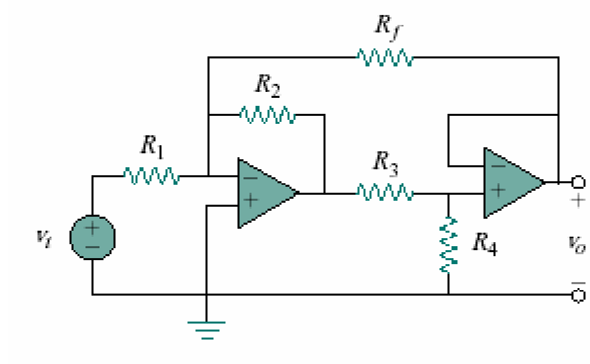


Figure 5.89

Chapter 5, Solution 62.

Let v_1 = output of the first op amp
v_2 = output of the second op amp

The first stage is a summer

$$v_1 = -\frac{R_2}{R_1}v_i - \frac{R_2}{R_f}v_o \qquad (1)$$

The second stage is a follower. By voltage division

$$v_o = v_2 = \frac{R_4}{R_3 + R_4}v_1 \longrightarrow v_1 = \frac{R_3 + R_4}{R_4}v_o \qquad (2)$$

From (1) and (2),

$$\left(1 + \frac{R_3}{R_4}\right)v_o = -\frac{R_2}{R_1}v_i - \frac{R_2}{R_f}v_o$$

$$\left(1 + \frac{R_3}{R_4} + \frac{R_2}{R_f}\right)v_o = -\frac{R_2}{R_1}v_i$$

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1}\cdot\frac{1}{1 + \frac{R_3}{R_4} + \frac{R_2}{R_f}} = \underline{\frac{-R_2R_4R_f}{R_1(R_2R_4 + R_3R_f + R_4R_f)}}$$

Chapter 5, Problem 63

Determine the gain v_o/v_i of the circuit in Fig. 5.90.

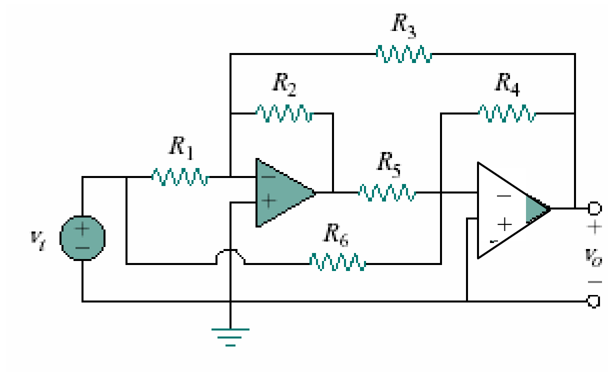


Figure 5.90

Chapter 5, Solution 63.

The two op amps are summers. Let v_1 be the output of the first op amp. For the first stage,

$$v_1 = -\frac{R_2}{R_1}v_i - \frac{R_2}{R_3}v_o \qquad (1)$$

For the second stage,

$$v_o = -\frac{R_4}{R_5}v_1 - \frac{R_4}{R_6}v_i \qquad (2)$$

Combining (1) and (2),

$$v_o = \frac{R_4}{R_5}\left(\frac{R_2}{R_1}\right)v_i + \frac{R_4}{R_5}\left(\frac{R_2}{R_3}\right)v_o - \frac{R_4}{R_6}v_i$$

$$v_o\left(1 - \frac{R_2R_4}{R_3R_5}\right) = \left(\frac{R_2R_4}{R_1R_5} - \frac{R_4}{R_6}\right)v_i$$

$$\frac{v_o}{v_i} = \frac{\frac{R_2R_4}{R_1R_5} - \frac{R_4}{R_6}}{1 - \frac{R_2R_4}{R_3R_5}}$$

Chapter 5, Problem 64

For the op amp circuit shown in Fig. 5.91, find v_o/v_s.

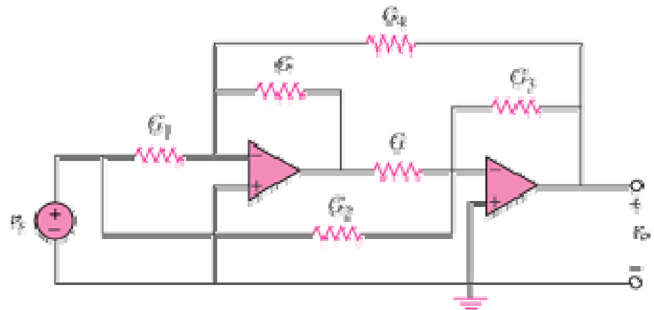
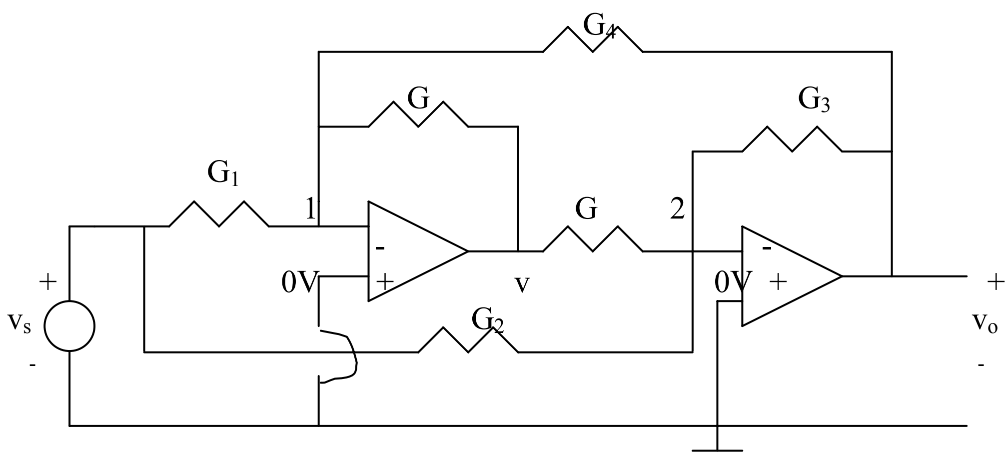


Figure 5.91

Chapter 5, Solution 64



At node 1, $v_1=0$ so that KCL gives

$$G_1 v_s + G_4 v_o = -Gv \quad (1)$$

At node 2,

$$G_2 v_s + G_3 v_o = -Gv \quad (2)$$

From (1) and (2),

$$G_1 v_s + G_4 v_o = G_2 v_s + G_3 v_o \longrightarrow (G_1 - G_2)v_s = (G_3 - G_4)v_o$$

or

$$\frac{v_o}{v_s} = \frac{G_1 - G_2}{G_3 - G_4}$$

Chapter 5, Problem 65

Find v_o in the op amp circuit of Fig. 5.92.

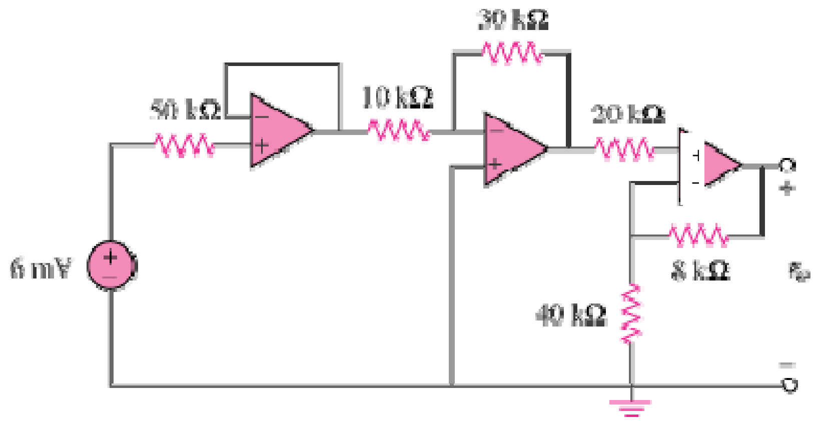


Figure 5.92

Chapter 5, Solution 65

The output of the first op amp (to the left) is 6 mV. The second op amp is an inverter so that its output is

$$v_o' = -\frac{30}{10}(6\text{mV}) = \text{-}18\,\text{mV}$$

The third op amp is a noninverter so that

$$v_o' = \frac{40}{40+8}v_o \quad \longrightarrow \quad v_o = \frac{48}{40}v_o' = \underline{-21.6\,\text{mV}}$$

Chapter 5, Problem 66

For the circuit in Fig. 5.93, find v_o.

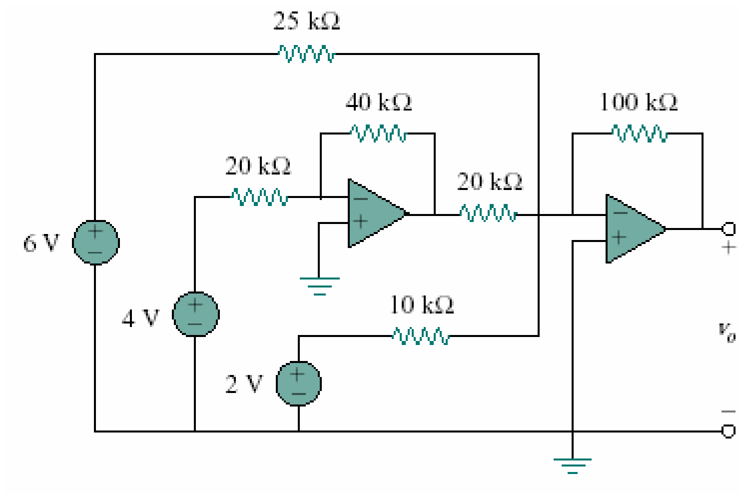


Figure 5.93

Chapter 5, Solution 66.

$$v_o = \frac{-100}{25}(6) - \frac{100}{20}\left(-\frac{40}{20}\right)(4) - \frac{100}{10}(2)$$
$$= -24 + 40 - 20 = \underline{\mathbf{-4V}}$$

Chapter 5, Problem 67

Obtain the output v_o in the circuit of Fig. 5.94.

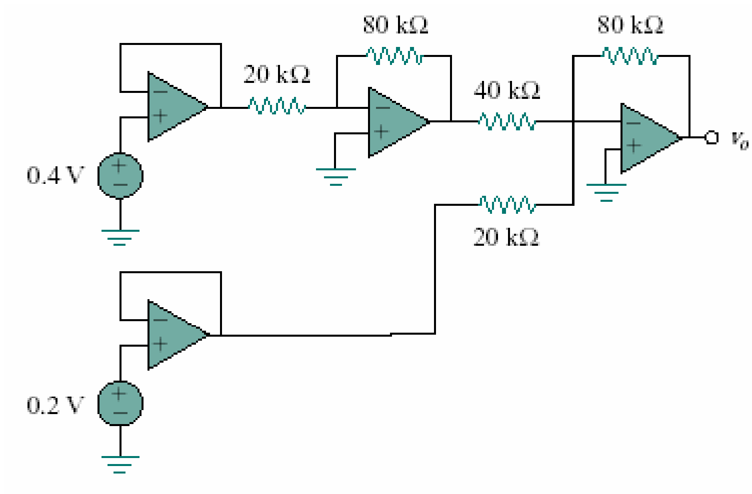


Figure 5.94

Chapter 5, Solution 67.

$$v_o = -\frac{80}{40}\left(-\frac{80}{20}\right)(0.2) - \frac{80}{20}(0.2)$$

$$= 3.2 - 0.8 = \underline{\mathbf{2.4V}}$$

Chapter 5, Problem 68.

Find v_o in the circuit in Fig. 5.95, assuming that $R_f = \infty$ (open circuit).

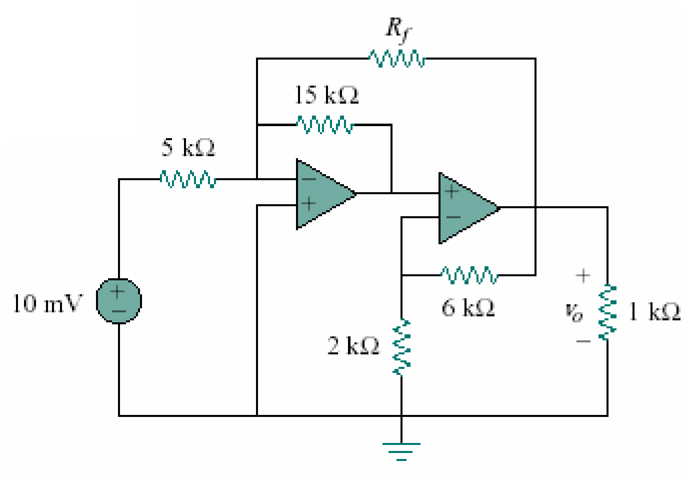


Figure 5.95

Chapter 5, Solution 68.

If $R_q = \infty$, the first stage is an inverter.

$$V_a = -\frac{15}{5}(10) = -30\text{mV}$$

when V_a is the output of the first op amp.

The second stage is a noninverting amplifier.

$$v_o = \left(1 + \frac{6}{2}\right)v_a = (1+3)(-30) = \underline{\mathbf{-120mV}}$$

Chapter 5, Problem 69

Repeat the previous problem if $R_f = 10\ \text{k}\Omega$.

5.68 Find v_o in the circuit in Fig. 5.93, assuming that $R_f = \infty$ (open circuit).

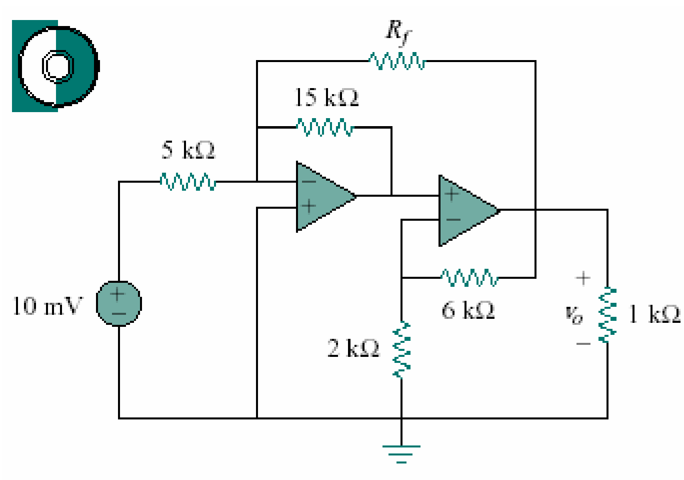


Figure 5.93

Chapter 5, Solution 69.

In this case, the first stage is a summer

$$v_a = -\frac{15}{5}(10) - \frac{15}{10}v_o = -30 - 1.5v_o$$

For the second stage,

$$v_o = \left(1 + \frac{6}{2}\right)v_a = 4v_a = 4(-30 - 1.5v_o)$$

$$7v_o = -120 \longrightarrow v_o = -\frac{120}{7} = \underline{\mathbf{-17.143mV}}$$

Chapter 5, Problem 70

Determine v_o in the op amp circuit of Fig. 5.96.

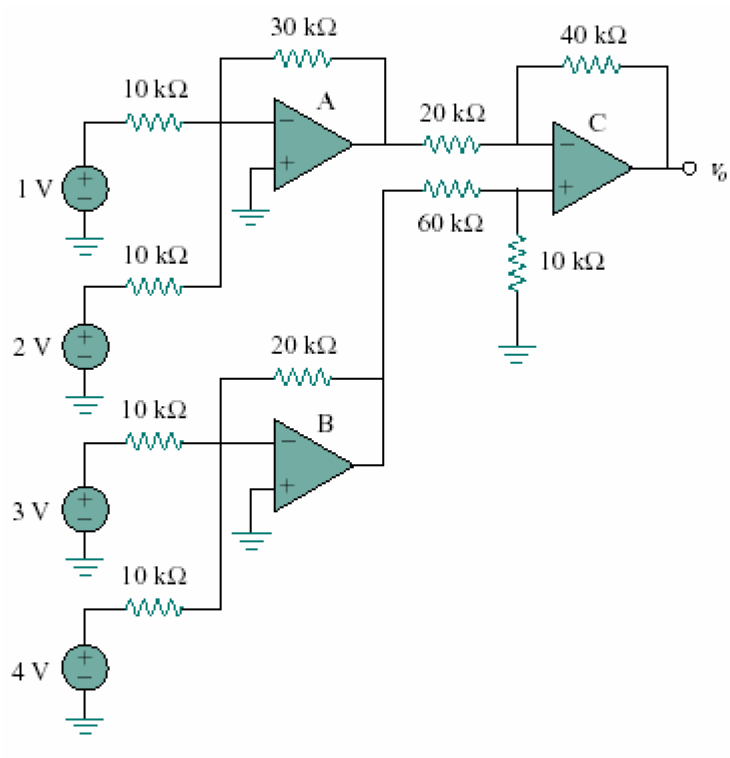


Figure 5.96

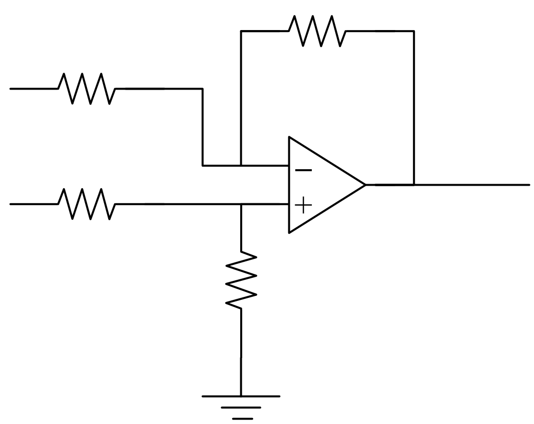
Chapter 5, Solution 70.

The output of amplifier A is

$$v_A = -\frac{30}{10}(1) - \frac{30}{10}(2) = -9$$

The output of amplifier B is

$$v_B = -\frac{20}{10}(3) - \frac{20}{10}(4) = -14$$



$$v_b = \frac{10}{60+10}(-14) = -2V$$

At node a, $\frac{v_A - v_a}{20} = \frac{v_a - v_o}{40}$

But $v_a = v_b$ = -2V, 2(-9+2) = -2-v_o

Therefore, v_o = **<u>12V</u>**

Chapter 5, Problem 71

Determine v_o in the op amp circuit in Fig. 5.97.

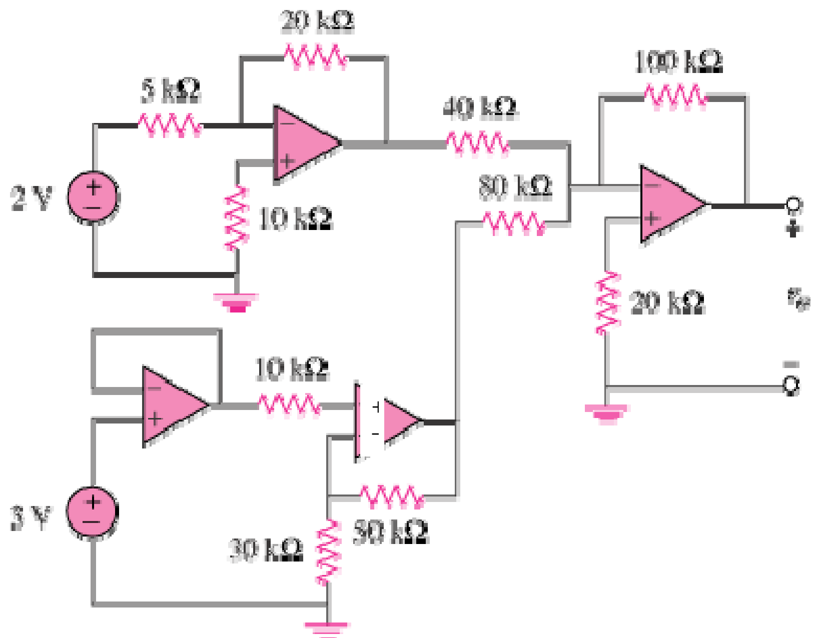
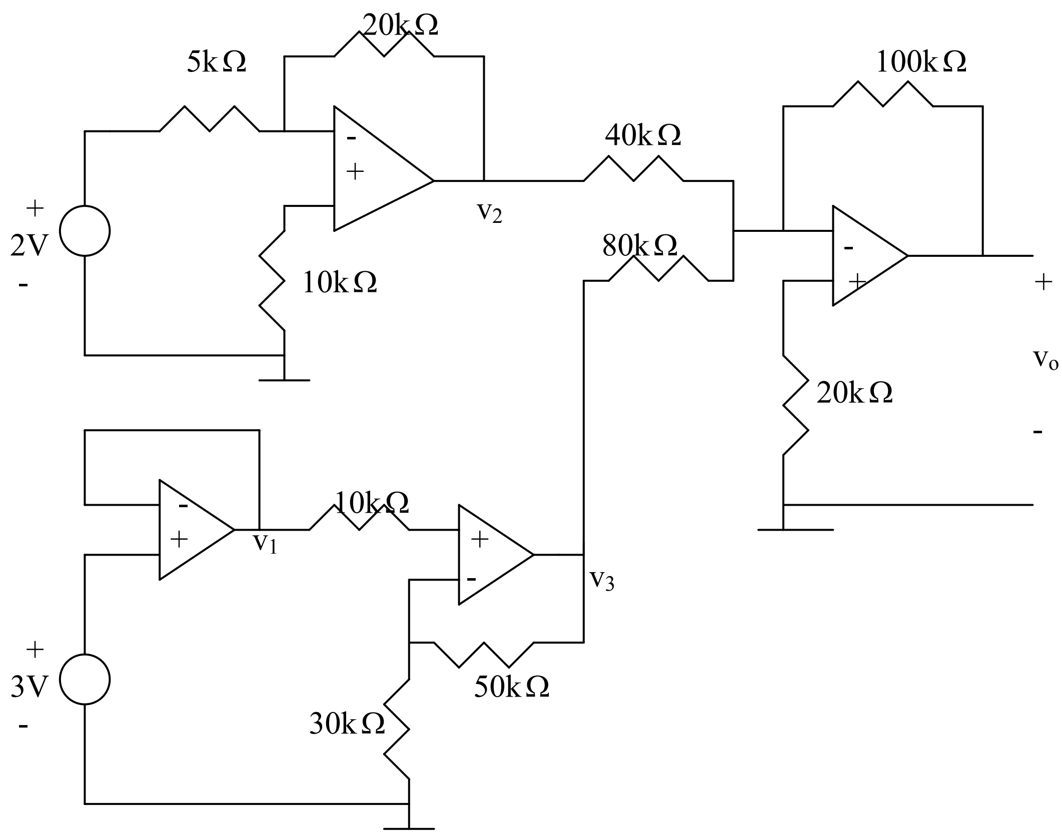


Figure 5.97

Chapter 5, Solution 71



$$v_1 = 3, \quad v_2 = -\frac{20}{5}(2) = -8, \quad v_3 = (1+\frac{50}{30})v_1 = 8$$

$$v_o = -\left(\frac{100}{40}v_2 + \frac{100}{80}v_3\right) = -(-20+10) = \underline{10\text{ V}}$$

Chapter 5, Problem 72

Find the load voltage v_L in the circuit of Fig. 5.98.

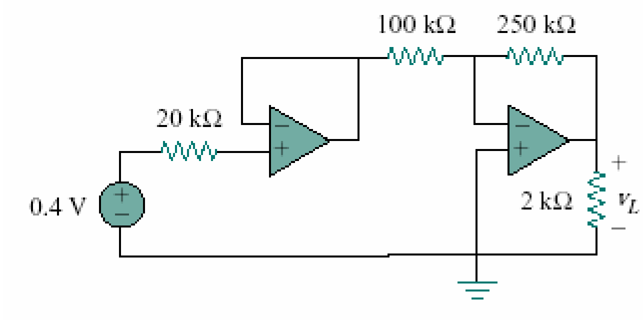


Figure 5.98

Chapter 5, Solution 72.

Since no current flows into the input terminals of ideal op amp, there is no voltage drop across the 20 kΩ resistor. As a voltage summer, the output of the first op amp is

$$v_{01} = 0.4$$

The second stage is an inverter

$$v_2 = -\frac{250}{100} v_{01}$$
$$= -2.5(0.4) = \underline{\mathbf{-1V}}$$

Chapter 5, Problem 73

Determine the load voltage v_L in the circuit of Fig. 5.99.

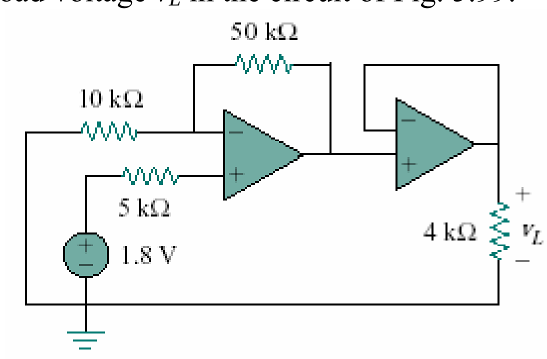


Figure 5.99

Chapter 5, Solution 73.

The first stage is an inverter. The output is

$$v_{01} = -\frac{50}{10}(-1.8) + 1.8 = 10.8\text{V}$$

The second stage is

$$v_2 = v_{01} = \underline{\mathbf{10.8V}}$$

Chapter 5, Problem 74

Find i_o in the op amp circuit of Fig. 5.100.

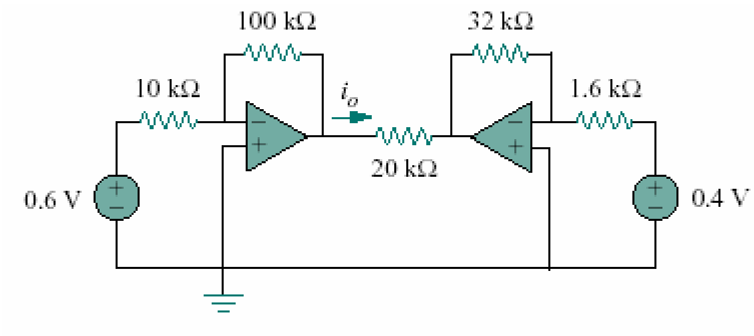


Figure 5.100

Chapter 5, Solution 74.

Let v_1 = output of the first op amp
v_2 = input of the second op amp.

The two sub-circuits are inverting amplifiers

$$v_1 = -\frac{100}{10}(0.6) = -6\text{V}$$

$$v_2 = -\frac{32}{1.6}(0.4) = -8\text{V}$$

$$i_o = \frac{v_1 - v_2}{20\text{k}} = -\frac{-6+8}{20\text{k}} = \underline{\mathbf{100\ \mu A}}$$

Chapter 5, Problem 75

Rework Example 5.11 using the nonideal op amp LM324 instead of uA741.

Example 5.11 - Use *PSpice* to solve the op amp circuit for Example 5.1.

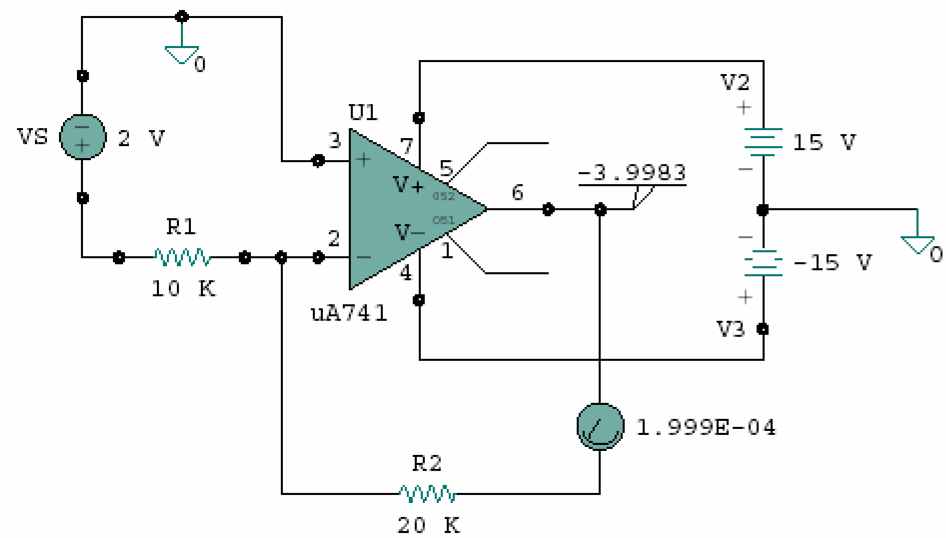


Figure 5.34 Schematic for Example 5.11.

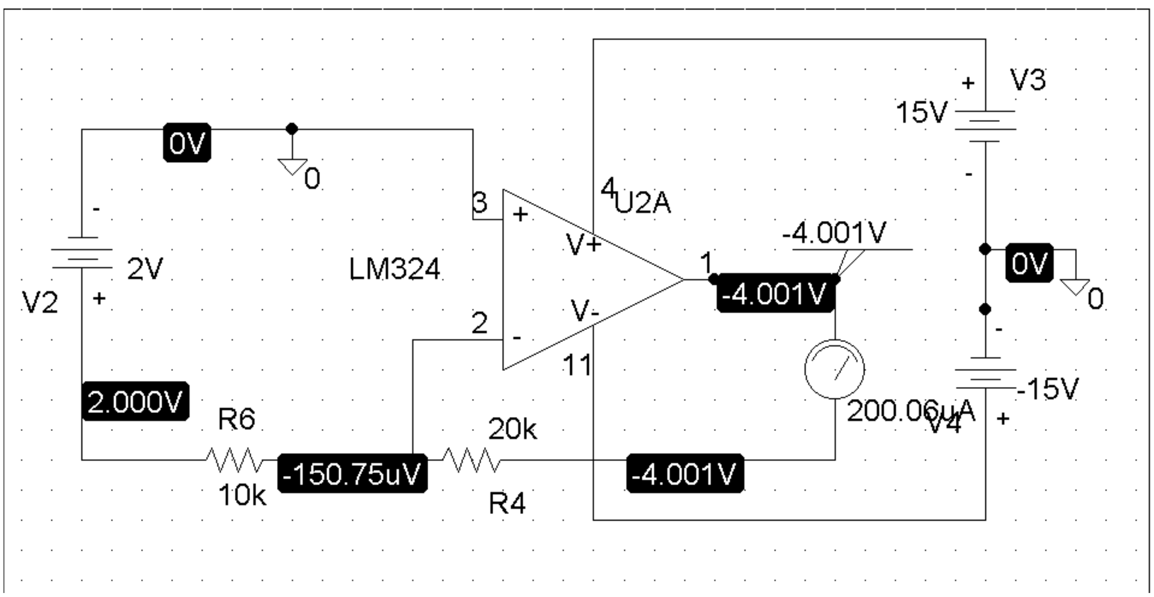
Chapter 5, Solution 75.

The schematic is shown below. Pseudo-components VIEWPOINT and IPROBE are involved as shown to measure v_o and i respectively. Once the circuit is saved, we click Analysis | Simulate. The values of v and i are displayed on the pseudo-components as:

$$i = \underline{\mathbf{200\ \mu A}}$$

$$(v_o/v_s) = -4/2 = \underline{-2}$$

The results are slightly different than those obtained in Example 5.11.



Chapter 5, Problem 76

Solve Prob. 5.19 using *PSpice* and op amp uA741.

5.19 Determine i_o in the circuit of Fig. 5.57.

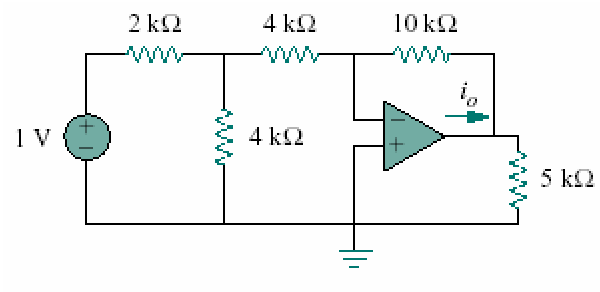
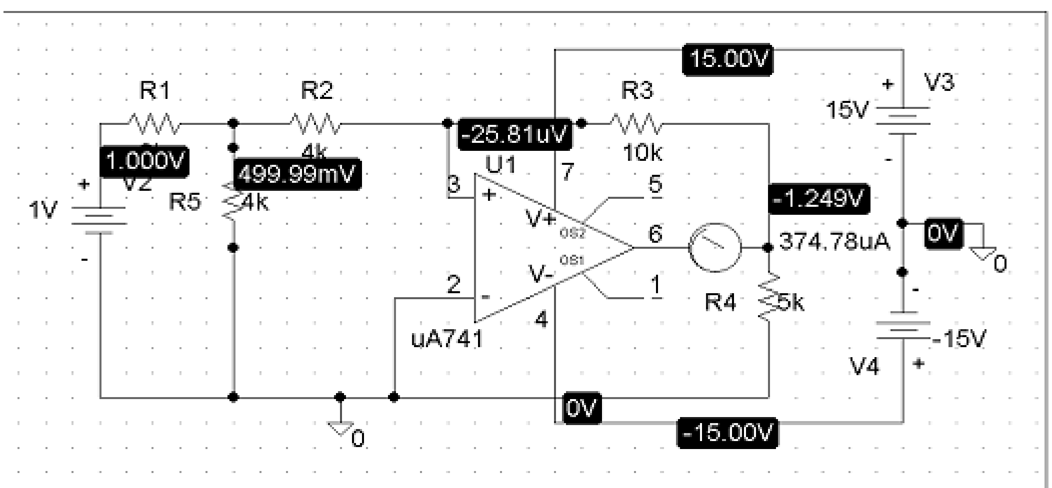


Figure 5.57

Chapter 5, Solution 76.

The schematic is shown below. IPROBE is inserted to measure i_o. Upon simulation, the value of i_o is displayed on IPROBE as

$$i_o = \underline{\mathbf{-374.78\ \mu A}}$$



Chapter 5, Problem 77

Solve Prob. 5.48 using *PSpice* and op amp LM324.

5.48 The circuit in Fig. 5.78 is a differential amplifier driven by a bridge. Find v_o.

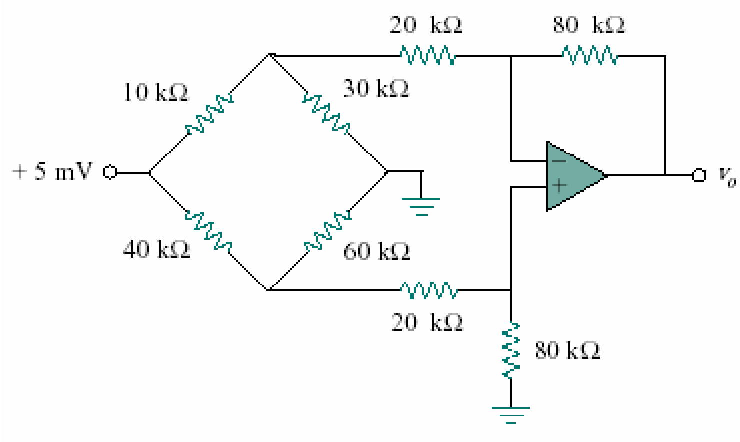
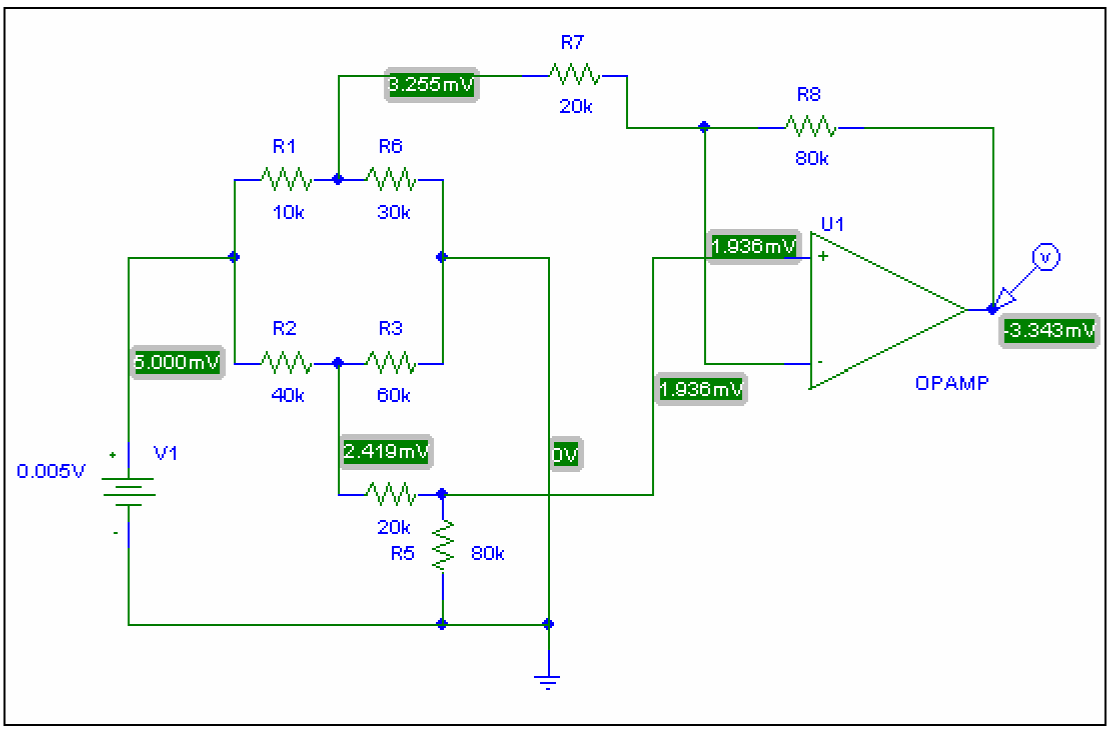


Figure 5.78

Chapter 5, Solution 77.

The schematic for the PSpice solution is shown below.

Note that the output voltage, **–3.343 mV**, agrees with the answer to problem, 5.48.



Chapter 5, Problem 78

Use *PSpice* to obtain v_o in the circuit of Fig. 5.101.

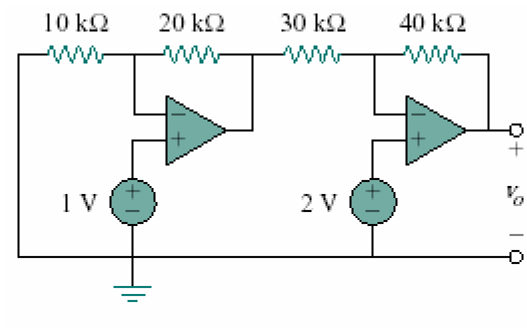
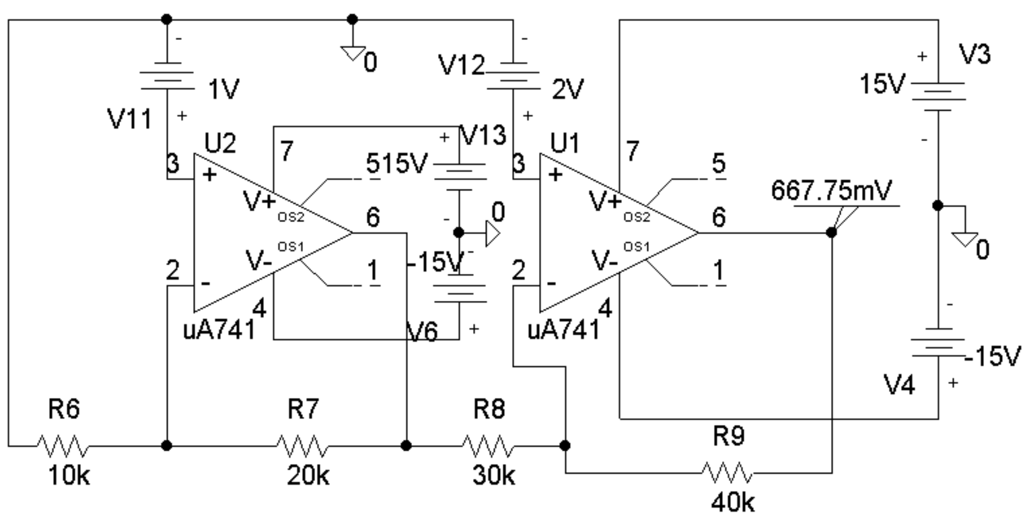


Figure 5.101

Chapter 5, Solution 78.

The circuit is constructed as shown below. We insert a VIEWPOINT to display v_o. Upon simulating the circuit, we obtain,

$$v_o = \underline{\mathbf{667.75\ mV}}$$



Chapter 5, Problem 79

Determine v_o in the op amp circuit of Fig. 5.102 using *PSpice*.

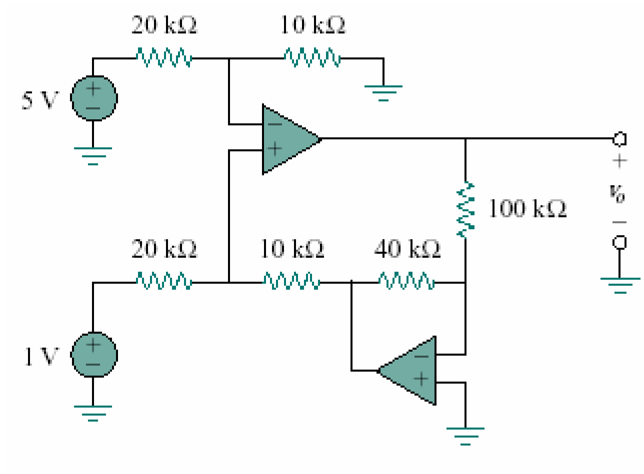
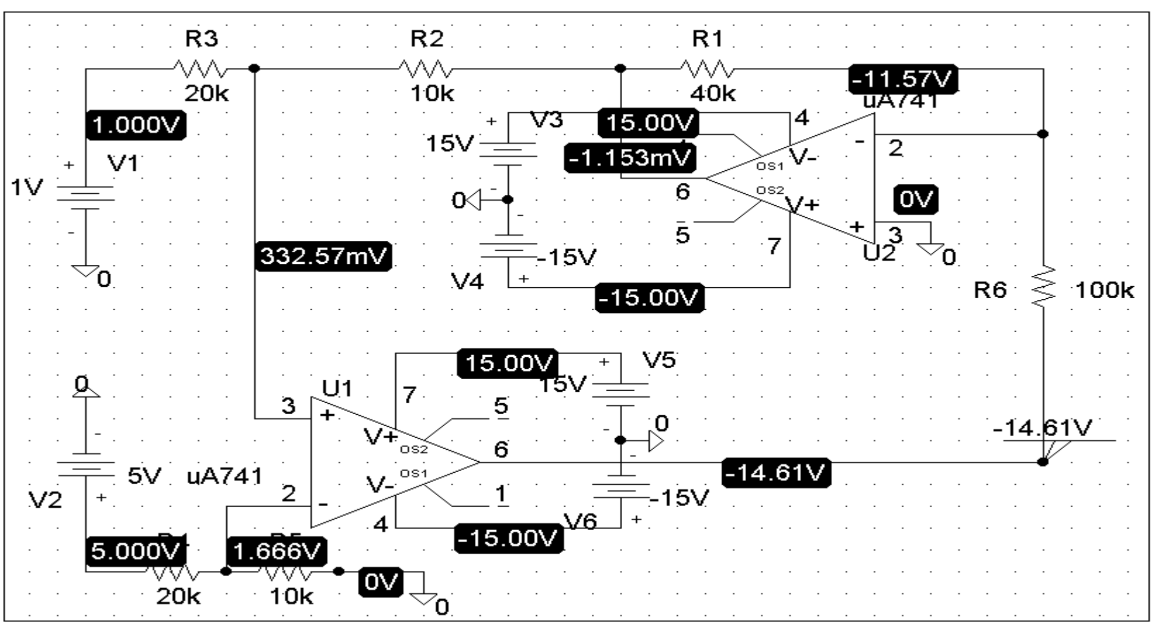


Figure 5.102

Chapter 5, Solution 79.

The schematic is shown below. A pseudo-component VIEWPOINT is inserted to display v_o. After saving and simulating the circuit, we obtain,

$$v_o = \underline{\mathbf{-14.61\ V}}$$



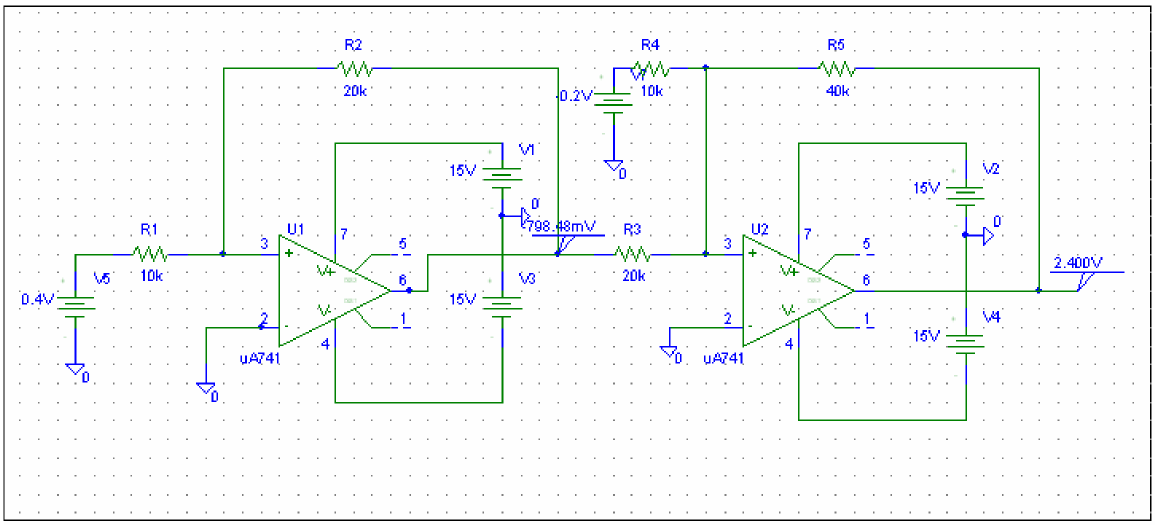
Chapter 5, Problem 80.

Use PSpice to solve Prob. 5.61.

Chapter 5, Solution 80.

The schematic is as shown below. After it is saved and simulated, we obtain

$$v_o = \underline{\mathbf{2.4\ V}}.$$



Chapter 5, Problem 81

Use *PSpice* to verify the results in Example 5.9. Assume nonideal op amps LM324.

Example 5.9 - Determine *vo* and *io* in the op amp circuit in Fig. 5.30.
Answer: 10 V, 1 mA.

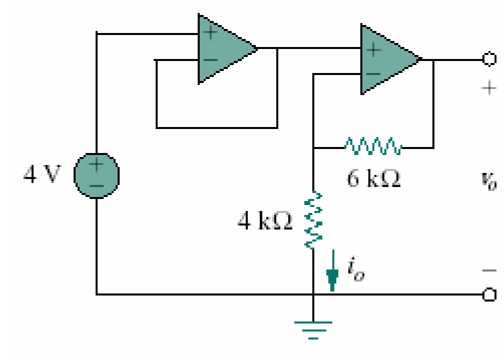


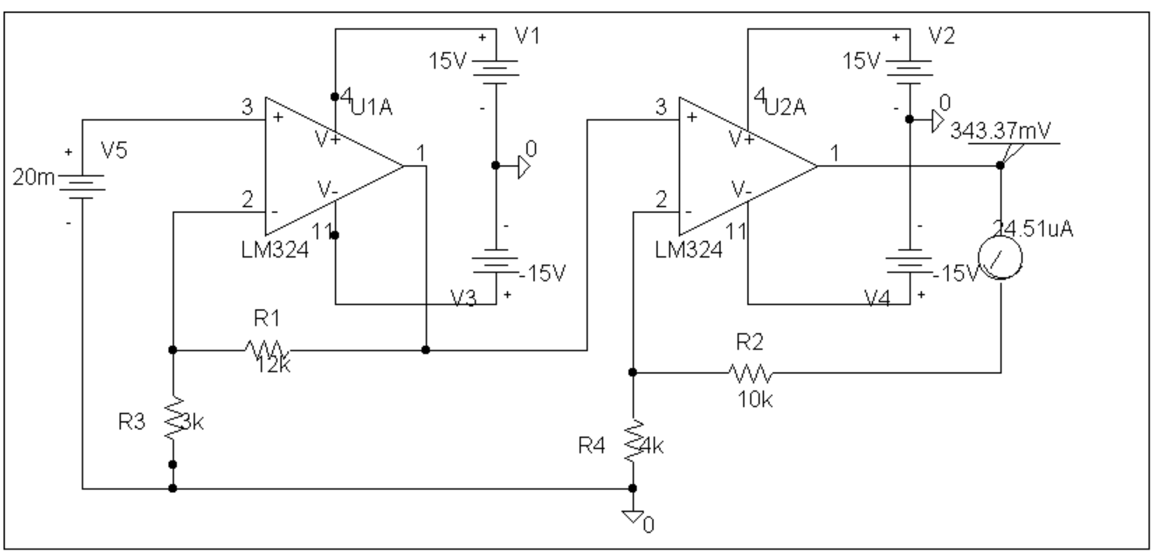
Figure 5.30 For Practice Prob. 5.9.

Chapter 5, Solution 81.

The schematic is shown below. We insert one VIEWPOINT and one IPROBE to measure v_o and i_o respectively. Upon saving and simulating the circuit, we obtain,

$$v_o = \underline{\mathbf{343.4\ mV}}$$

$$i_o = \underline{\mathbf{24.51\ \mu A}}$$



Chapter 5, Problem 82

A five-bit DAC covers a voltage range of 0 to 7.75 V. Calculate how much voltage each bit is worth.

Chapter 5, Solution 82.

The maximum voltage level corresponds to

$$11111 = 2^5 - 1 = 31$$

Hence, each bit is worth $(7.75/31) = \underline{\mathbf{250\ mV}}$

Chapter 5, Problem 83

Design a six-bit digital-to-analog converter.

(a) If $|V_o| = 1.1875$ V is desired, what should $[V_1V_2V_3V_4V_5V_6]$ be?
(b) Calculate $|V_o|$ if $[V_1V_2V_3V_4V_5V_6] = [011011]$.
(c) What is the maximum value $|V_o|$ can assume?

Chapter 5, Solution 83.

The result depends on your design. Hence, let R_G = 10 k ohms, R_1 = 10 k ohms, R_2 = 20 k ohms, R_3 = 40 k ohms, R_4 = 80 k ohms, R_5 = 160 k ohms, R_6 = 320 k ohms, then,

$$-v_o = (R_f/R_1)v_1 + \text{---------} + (R_f/R_6)v_6$$

$$= v_1 + 0.5v_2 + 0.25v_3 + 0.125v_4 + 0.0625v_5 + 0.03125v_6$$

(a) $|v_o| = 1.1875 = 1 + 0.125 + 0.0625 = 1 + (1/8) + (1/16)$ which implies,

$$[v_1\ v_2\ v_3\ v_4\ v_5\ v_6] = \underline{\mathbf{[100110]}}$$

(b) $|v_o| = 0 + (1/2) + (1/4) + 0 + (1/16) + (1/32) = (27/32) = \underline{\mathbf{843.75\ mV}}$

(c) This corresponds to [1 1 1 1 1 1].

$|v_o| = 1 + (1/2) + (1/4) + (1/8) + (1/16) + (1/32) = 63/32 = \underline{\mathbf{1.96875\ V}}$

Chapter 5, Problem 84

A four-bit *R-2R ladder* DAC is presented in Fig. 5.103.

(a) Show that the output voltage is given by

$$-V_o = R_f\left(\frac{V_1}{2R}+\frac{V_2}{4R}+\frac{V_3}{8R}+\frac{V_4}{16R}\right)$$

(b) If $R_f = 12\text{ k}\Omega$ and $R = 10\text{ k}\Omega$, find $|V_o|$ for $[V_1V_2V_3V_4] = [1011]$ and $[V_1V_2V_3V_4] = [0101]$.

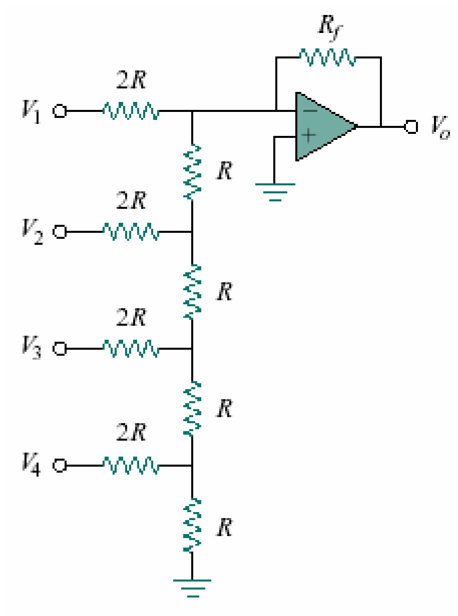
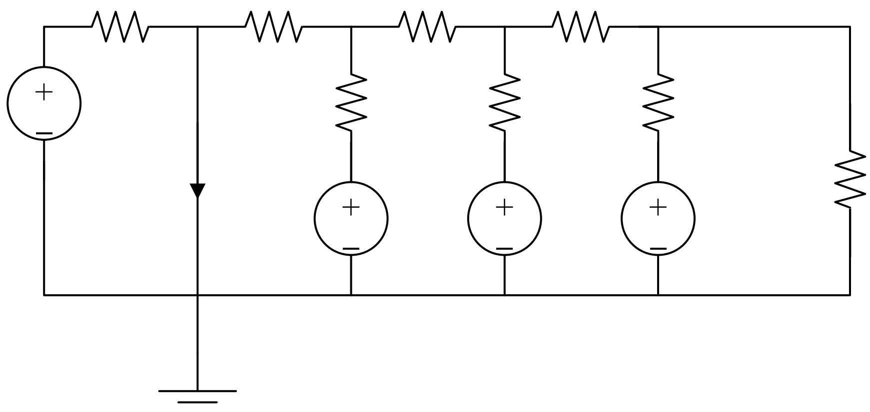


Figure 5.103

Chapter 5, Solution 84.

For (a), the process of the proof is time consuming and the results are only approximate, but close enough for the applications where this device is used.

(a) The easiest way to solve this problem is to use superposition and to solve for each term letting all of the corresponding voltages be equal to zero. Also, starting with each current contribution (i_k) equal to one amp and working backwards is easiest.



For the first case, let $v_2 = v_3 = v_4 = 0$, and $i_1 = 1A$.

Therefore, $v_1 = 2R$ volts or $i_1 = v_1/(2R)$.

Second case, let $v_1 = v_3 = v_4 = 0$, and $i_2 = 1A$.

Therefore, $v_2 = 85R/21$ volts or $i_2 = 21v_2/(85R)$. Clearly this is not ($1/4^{th}$), so where is the difference? $(21/85) = 0.247$ which is a really good approximation for 0.25. Since this is a practical electronic circuit, the result is good enough for all practical purposes.

Now for the third case, let $v_1 = v_2 = v_4 = 0$, and $i_3 = 1A$.

Therefore, $v_3 = 8.5R$ volts or $i_3 = v_3/(8.5R)$. Clearly this is not $(1/8^{th})$, so where is the difference? $(1/8.5) = 0.11765$ which is a really good approximation for 0.125. Since this is a practical electronic circuit, the result is good enough for all practical purposes.

Finally, for the fourth case, let $v_1 = v_2 = v_4 = 0$, and $i_3 = 1A$.

Therefore, $v_4 = 16.25R$ volts or $i_4 = v_4/(16.25R)$. Clearly this is not $(1/16^{th})$, so where is the difference? $(1/16.25) = 0.06154$ which is a really good approximation for 0.0625. Since this is a practical electronic circuit, the result is good enough for all practical purposes.

Please note that a goal of a lot of electronic design is to come up with practical circuits that are economical to design and build yet give the desired results.

(b) If $R_f = 12$ k ohms and $R = 10$ k ohms,

$$-v_o = (12/20)[v_1 + (v_2/2) + (v_3/4) + (v_4/8)]$$

$$= 0.6[v_1 + 0.5v_2 + 0.25v_3 + 0.125v_4]$$

For $[v_1\ v_2\ v_3\ v_4] = [1\ 0\ 11]$,

$$|v_o| = 0.6[1 + 0.25 + 0.125] = \underline{\mathbf{825\ mV}}$$

For $[v_1\ v_2\ v_3\ v_4] = [0\ 1\ 0\ 1]$,

$$|v_o| = 0.6[0.5 + 0.125] = \underline{\mathbf{375\ mV}}$$

Chapter 5, Problem 85.

In the op amp circuit of Fig. 5.104, find the value of R so that the power absorbed by the 10-kΩ resistor is 10 mW. Take $v_s = 2V$.

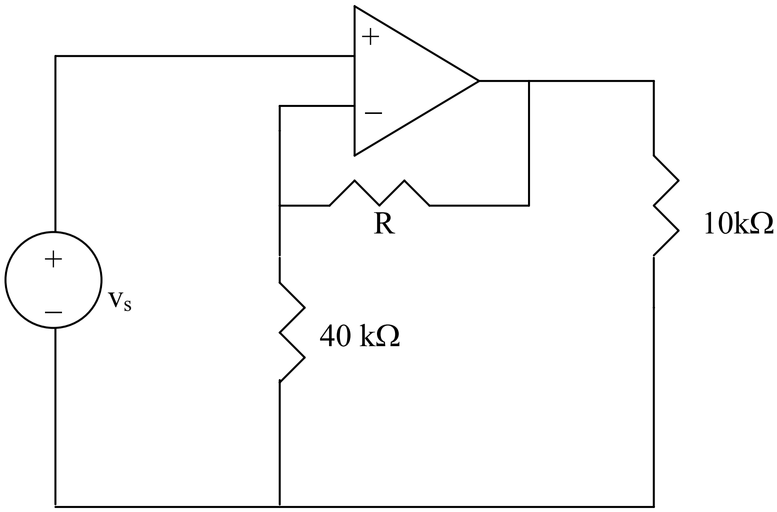


Figure 5.104 For Prob. 5.85.

Chapter 5, Solution 85.

This is a noninverting amplifier.

$$v_o = (1 + R/40k)v_s = (1 + R/40k)2$$

The power being delivered to the 10-kΩ give us

$$P = 10 \text{ mW} = (v_o)^2/10k \quad \text{or} \quad v_o = \sqrt{10^{-2}x10^4} = 10V$$

Returning to our first equation we get

$$10 = (1 + R/40k)2 \quad \text{or} \quad R/40k = 5 - 1 = 4$$

Thus, R = **<u>160 kΩ</u>**.

Chapter 5, Problem 86

Assuming a gain of 200 for an IA, find its output voltage for:
(a) $v_1 = 0.402$ V and $v_2 = 0.386$ V
(b) $v_1 = 1.002$ V and $v_2 = 1.011$ V.

Chapter 5, Solution 86.

$$v_o = A(v_2 - v_1) = 200(v_2 - v_1)$$

(a) $$v_o = 200(0.386 - 0.402) = \underline{\mathbf{-3.2\ V}}$$

$$v_o = 200(1.011 - 1.002) = \underline{\mathbf{1.8\ V}}$$

Chapter 5, Problem 87

Figure 5.105 displays a two-op-amp instrumentation amplifier. Derive an expression for v_o in terms of v_1 and v_2. How can this amplifier be used as a subtractor?

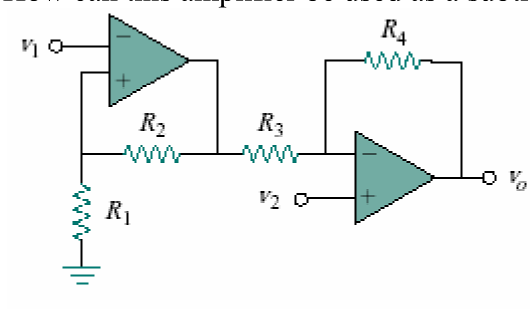


Figure 5.105

Chapter 5, Solution 87.

The output, v_a, of the first op amp is,

$$v_a = (1 + (R_2/R_1))v_1 \qquad (1)$$

Also, $$v_o = (-R_4/R_3)v_a + (1 + (R_4/R_3))v_2 \qquad (2)$$

Substituting (1) into (2),

$$v_o = (-R_4/R_3)\,(1 + (R_2/R_1))v_1 + (1 + (R_4/R_3))v_2$$

Or, $$v_o = \underline{\mathbf{(1 + (R_4/R_3))v_2 - (R_4/R_3 + (R_2R_4/R_1R_3))v_1}}$$

If $R_4 = R_1$ and $R_3 = R_2$, then,

$$v_o = (1 + (R_4/R_3))(v_2 - v_1)$$

which is a subtractor with a gain of $\underline{\mathbf{(1 + (R_4/R_3))}}$.

Chapter 5, Problem 88

Figure 5.106 shows an instrumentation amplifier driven by a bridge. Obtain the gain v_o/v_i of the amplifier.

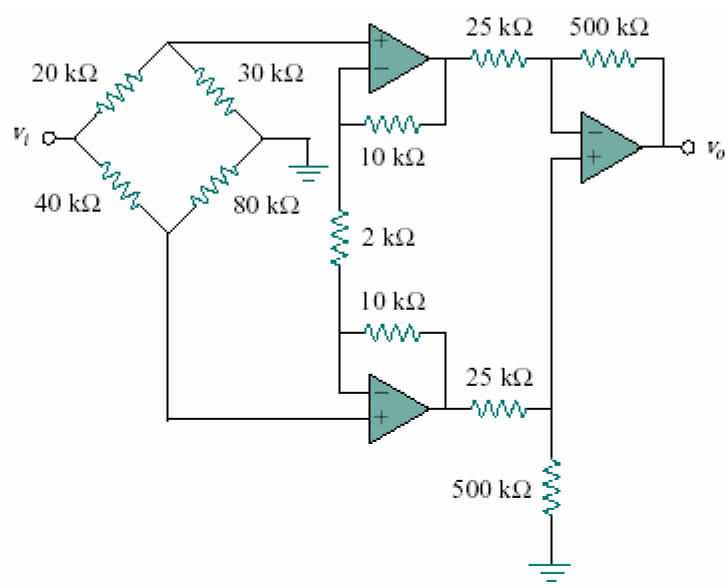


Figure 5.106

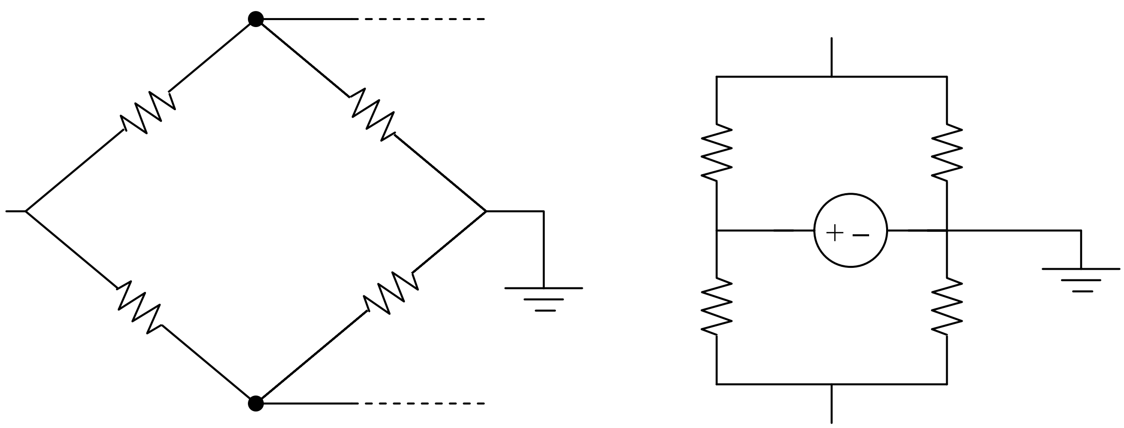
Chapter 5, Solution 88.

We need to find V_{Th} at terminals a – b, from this,

$$v_o = (R_2/R_1)(1 + 2(R_3/R_4))V_{Th} = (500/25)(1 + 2(10/2))V_{Th}$$

$$= 220V_{Th}$$

Now we use Fig. (b) to find V_{Th} in terms of v_i.



$$v_a = (3/5)v_i, \quad v_b = (2/3)v_i$$

$$V_{Th} = v_b – v_a \ (1/15)v_i$$

$$(v_o/v_i) = A_v = -220/15 = \underline{\mathbf{-14.667}}$$

Chapter 5, Problem 89.

Design a circuit that provides a relationship between output voltage v_o and input voltage v_s such that $v_o = 12v_s - 10$. Two op amps, a 6-V battery and several resistors are available.

Chapter 5, Solution 89.

A **summer** with $\mathbf{v_o = -v_1 - (5/3)v_2}$ where v_2 = **6-V battery** and an **inverting amplifier** with $\mathbf{v_1 = -12v_s}$.

Chapter 5, Problem 90

The op amp circuit in Fig. 5.107 is a *current amplifier*. Find the current gain i_o/i_s of the amplifier.

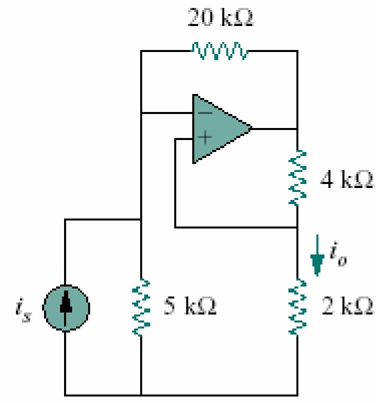
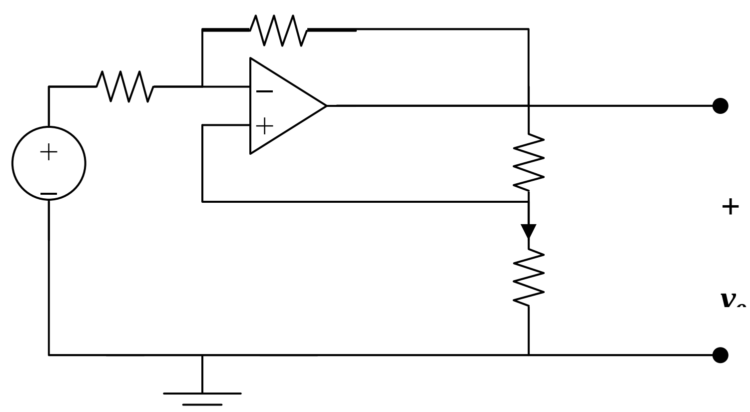


Figure 5.107

Chapter 5, Solution 90.

Transforming the current source to a voltage source produces the circuit below,

At node b, $v_b = (2/(2 + 4))v_o = v_o/3$



At node a, $(5i_s – v_a)/5 = (v_a – v_o)/20$

But $v_a = v_b = v_o/3$. $20i_s – (4/3)v_o = (1/3)v_o – v_o$, or $i_s = v_o/30$

$i_o = [(2/(2 + 4))/2]v_o = v_o/6$

$i_o/i_s = (v_o/6)/(v_o/30) = \underline{\mathbf{5}}$

Chapter 5, Problem 91

A noninverting current amplifier is portrayed in Fig. 5.108. Calculate the gain i_o/i_s. Take R_1 = 8 kΩ and R_2 = 1 kΩ.

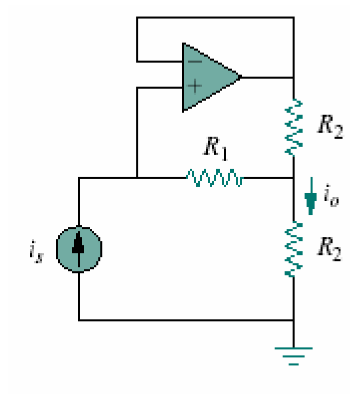
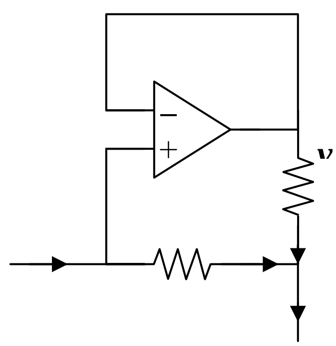


Figure 5.108

Chapter 5, Solution 91.



$$i_o = i_1 + i_2 \qquad (1)$$

But $$i_1 = i_s \qquad (2)$$

R_1 and R_2 have the same voltage, v_o, across them.

$$R_1 i_1 = R_2 i_2, \text{ which leads to } i_2 = (R_1/R_2) i_1 \qquad (3)$$

Substituting (2) and (3) into (1) gives,

$$i_o = i_s(1 + R_1/R_2)$$

$$i_o/i_s = 1 + (R_1/R_2) = 1 + 8/1 = \underline{\mathbf{9}}$$

Chapter 5, Problem 92

Refer to the *bridge amplifier* shown in Fig. 5.109. Determine the voltage gain v_o/v_i .

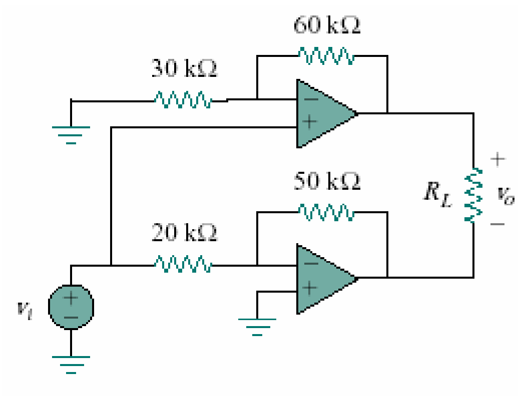


Figure 5.109

Chapter 5, Solution 92

The top op amp circuit is a non-inverter, while the lower one is an inverter. The output at the top op amp is

$$v_1 = (1 + 60/30)v_i = 3v_i$$

while the output of the lower op amp is

$$v_2 = -(50/20)v_i = -2.5v_i$$

Hence, $$v_o = v_1 - v_2 = 3v_i + 2.5v_i = 5.5v_i$$

$$v_o/v_i = \underline{\mathbf{5.5}}$$

Chapter 5, Problem 93

A voltage-to-current converter is shown in Fig. 5.110, which means that $i_L = Av_i$ if $R_1R_2 = R_3R_4$. Find the constant term A.

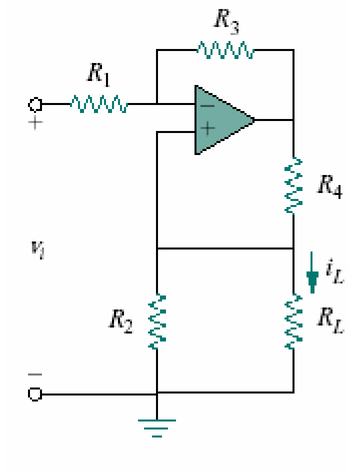
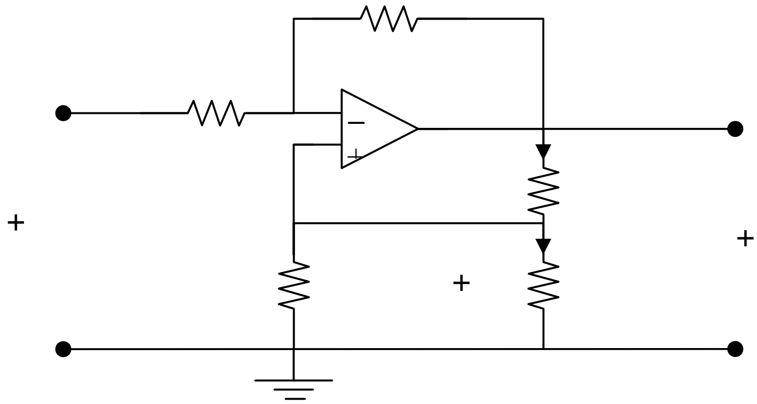


Figure 5.110

Chapter 5, Solution 93.



At node a, $(v_i - v_a)/R_1 = (v_a - v_o)/R_3$

$v_i - v_a = (R_1/R_2)(v_a - v_o)$

$v_i + (R_1/R_3)v_o = (1 + R_1/R_3)v_a$ (1)

But $v_a = v_b = v_L$. Hence, (1) becomes
$v_i = (1 + R_1/R_3)v_L - (R_1/R_3)v_o$ (2)

$i_o = v_o/(R_4 + R_2||R_L)$, $i_L = (R_2/(R_2 + R_L))i_o = (R_2/(R_2 + R_L))(v_o/(R_4 + R_2||R_L))$

Or, $v_o = i_L[(R_2 + R_L)(R_4 + R_2||R_L)/R_2$ (3)

But, $v_L = i_LR_L$ (4)

Substituting (3) and (4) into (2),
$v_i = (1 + R_1/R_3)\, i_LR_L - R_1[(R_2 + R_L)/(R_2R_3)](R_4 + R_2||R_L)i_L$

$= [((R_3 + R_1)/R_3)R_L - R_1((R_2 + R_L)/(R_2R_3)(R_4 + (R_2R_L/(R_2 + R_L))]i_L$

$= (1/A)i_L$

Thus,

$$A = \frac{\mathbf{1}}{\left(\mathbf{1}+\dfrac{\mathbf{R_1}}{\mathbf{R_3}}\right)\mathbf{R_L} - \mathbf{R_1}\left(\dfrac{\mathbf{R_2}+\mathbf{R_L}}{\mathbf{R_2R_3}}\right)\left(\mathbf{R_4}+\dfrac{\mathbf{R_2R_L}}{\mathbf{R_2}+\mathbf{R_L}}\right)}$$

Please note that A has the units of mhos. An easy check is to let every resistor equal 1-ohm and v_i equal to one amp. Going through the circuit produces $i_L = 1A$. Plugging into the above equation produces the same answer so the answer does check.